Two-Polarization Generalized Zero-Forcing Equalization for Channel Interference Suppression

by

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A Thesis Submitted in Partial Fulfillment of
the Requirements for the Degree of

Master of Science in Engineering

in the Department of Electrical and Computer Engineering

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This thesis is accepted.

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THE UNIVERSITY OF NEW BRUNSWICK

January, 2004

This thesis is dedicated to my parents,

Zongjun Jin and Wenzhe Li.

Abstract

The purpose of the research is to obtain results toward effective methods for combating the deleterious effects of various impairments arising in digital data transmission over a dually polarized fiber-optic communication channel.

The improvements in spectral efficiency can cause more co-channel interference (CCI) and adjacent-channel interference (ACI). Moreover, the transmission of M-state Quadrature Amplitude Modulated (QAM) signals via orthogonally polarized carriers is an effective method for reusing existing bandwidth with the advantage of more system capacity and a higher bit rate. However, the obstacle in the way of realizing these advantages is the unavoidable presence of cross-polarization interference (CPI) between two polarized signals.

A design possibility utilizing channel equalization and wide relative transmitter bandwidths to suppress intersymbol interference (ISI), CCI, ACI, CPI and noise is presented in this thesis and the existence of such equalization and the conditions to suppress all ISI, CCI, ACI and CPI are studied.

Acknowledgements

I wish to gratefully acknowledge the contributions made by various individuals to the completion of this thesis. First, and foremost, a special thank goes to my supervisor, Dr. Brent R. Petersen, who encouraged and supported me throughout entire research. Needless to say, without his help, this thesis would have never been completed.

The faculty of Department of Electrical and Computer Engineering, the staff and students of University of New Brunswick provided me support during the development of the thesis. I am grateful to have been associated with such great people.

I thank Zongjun Jin and Wenzhe Li, my parents, Hsueh-Ching Wu, my wife, Tzu-Wei Wu and Yun-Hsuan Lai, my in-laws for their love and support.

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List of Symbols

$a_{0h}(t)$	the input to the transmitter filter in the horizontal channel of the data transmitted. See equation (2.15).
$a_{0v}(t)$	the input to the transmitter filter in the vertical channel of the data transmitted. See equation (2.15).
$a_{Lh}(t)$	the input to the transmitter filter in the horizontal channel of the co-channel. See equation (2.17).
$a_{Lv}(t)$	the input to the transmitter filter in the vertical channel of the co-channel. See equation (2.17).
B_r	the normalized receiver bandwidth. See equation (4.4).
B_{ri}	the index factor. See equation (4.48).
B_t	the normalized transmitter bandwidth. See equation (4.2).
C	the normalized carrier spacing. See equation (4.6).
$\boldsymbol{c}_b(t)$	the complex baseband channel impulse response matrix. See equation (2.13).
$c_{bhh}(t)$	the baseband horizontal-to-horizontal channel impulse response. See equation (2.4).
$c_{bhv}(t)$	the baseband horizontal-to-vertical channel impulse response. See equation (2.4).
$c_{bvh}(t)$	the baseband vertical-to-horizontal channel impulse response. See equation (2.4).

$c_{bvv}(t)$ the baseband vertical-to-vertical channel impulse response. See equation (2.4).

$\boldsymbol{c}_p(t)$ the passband channel impulse response matrix. See equation (2.3).

$c_{phh}(t)$ the passband horizontal-to-horizontal channel impulse response. See equation (2.3).

$c_{phv}(t)$ the passband horizontal-to-vertical channel impulse response. See equation (2.3).

$c_{pvh}(t)$ the passband vertical-to-horizontal channel impulse response. See equation (2.3)

$c_{pvv}(t)$ the passband vertical-to-vertical channel impulse response. See equation (2.3).

$d_h[n]$ the output signal of the linear equalizer of the horizontal channel. See Fig. 2.4.

$d_v[n]$ the output signal of the linear equalizer of the vertical channel. See Fig. 2.4.

f the dummy parameter used as frequency index.

f_c the carrier frequency. See equation (2.2) and Fig. 2.1.

$\boldsymbol{h}_0(t)$ the equalized combined channel impulse response for the data transmitted. See equation (3.1).

$\boldsymbol{H}_0(f)$ the frequency response of the equalized combined channel for the data transmitted. See equation (3.3).

$H_{0hh}(f)$ the horizontal-to-horizontal frequency response of the equalized combined channel. See equation (3.7).

$h_{0hh}(t)$ the horizontal-to-horizontal impulse response of the equalized combined channel. See equation (3.11).

$H_{0hv}(f)$ the horizontal-to-vertical frequency response of the equalized combined channel. See equation (3.7).

$h_{0hv}(t)$ the horizontal-to-vertical impulse response of the equalized combined channel. See equation (3.11)

$H_{0vh}(f)$ the vertical-to-horizontal channel frequency response of the equalized combined channel. See equation (3.7).

$h_{0vh}(t)$ the vertical-to-horizontal channel impulse response of the equalized combined channel. See equation (3.11).

$H_{0vv}(f)$ the vertical-to-vertical channel frequency response of the equalized combined channel. See equation (3.7).

$h_{0vv}(t)$ the vertical-to-vertical channel impulse response of the equalized combined channel. See equation (3.11).

$\boldsymbol{h}_L(t)$ the equalized combined co-channels impulse responses. See equation (3.2).

$\boldsymbol{H}_L(f)$ the frequency response matrix of the equalized combined co-channel. See equation (3.4).

$H_{Lhh}(f)$ the horizontal-to-horizontal frequency response of the L^{th} equalized combined co-channels. See equation (3.10).

$h_{Lhh}(t)$ the horizontal-to-horizontal impulse response of the L^{th} equalized combined co-channel. See equation (3.12).

$H_{Lhv}(f)$ the horizontal-to-vertical frequency response of the L^{th} equalized combined co-channel. See equation (3.10).

$h_{Lhv}(t)$ the horizontal-to-vertical impulse response of the L^{th} equalized combined co-channel. See equation (3.12).

$H_{Lvh}(f)$ the vertical-to-horizontal channel frequency response of the L^{th} equalized combined co-channel. See equation (3.10).

$h_{Lvh}(t)$ the vertical-to-horizontal channel impulse response of the L^{th} equalized combined co-channel. See equation (3.12).

$H_{Lvv}(f)$ the vertical-to-vertical channel frequency response of the L^{th} equalized combined co-channel. See equation (3.10).

$h_{Lvv}(t)$ the vertical-to-vertical channel impulse response of the L^{th} equalized combined co-channel. See equation (3.12).

$\text{int}(\bullet)$ the integer part of $\bullet$. See equation (4.47).

k the dummy parameter used as a general index.

K_c an element of a set of real numbers. See equation (4.5).

K_r an element of a set of real numbers. See equation (4.3).

K_t an element of a set of real numbers. See equation (4.1)

L the number of interfering co-channels or adjacent channels. It is an element of a set of natural numbers, $\{1,2,3,\cdots\}$.

M an element of a set of real numbers. See equation (3.106).

$\min(\bullet, \circ)$ the minimum value among $\bullet$ and $\circ$. See equation (4.50).

n the dummy parameter used as general index.

N_{aci} the number of interfering ACI signals. See equation (4.47).

$\boldsymbol{n}_b(t)$ the baseband channel white noise matrix. See equation (2.14).

$n_{bh}(t)$ the baseband channel white noise in the horizontal channel. See equation (2.14).

$n_{bv}(t)$ the baseband channel white noise in the vertical channel.

N_E the number of equations. See equation (4.49).

$\boldsymbol{n}_p(t)$ the passband channel white noise vector. See equation (2.6).

$n_{ph}(t)$ the passband channel white noise in the horizontal channel. See equation (2.6).

$n_{pv}(t)$ the passband channel white noise in the vertical channel. See equation (2.6).

N_u the number of unknowns. See equation (4.50).

$p_0(t)$ the transmitter impulse response for the data transmitted. See equation (2.16).

$p_L(t)$ the transmitter impulse response for the L^{th} co-channel. See equation (2.18).

$\boldsymbol{r}(t)$ the impulse response matrix of the equalizer. See equation (2.22).

$\boldsymbol{R}(f)$ the frequency response matrix of the equalizer. See equation (2.25).

Re[•] the real part of •. See equation (2.24).

$R_{hh}(f)$ the receiver horizontal-to-horizontal channel frequency response. See equation (2.25).

$r_{hh}(t)$ the horizontal-to-horizontal impulse response of the equalizer. See equation (2.22).

$R_{hv}(f)$ the receiver horizontal-to-vertical channel frequency response. See equation (2.25).

$r_{hv}(t)$ the horizontal-to-vertical impulse response of the equalizer. See equation (2.22).

$R_{vh}(f)$ the receiver vertical-to-horizontal channel frequency response. See equation (2.25).

$r_{vh}(t)$ the vertical-to-horizontal impulse response of the equalizer. See equation (2.22).

$R_{vv}(f)$ the receiver vertical-to-vertical channel frequency response. See equation (2.25).

$r_{vv}(t)$ the vertical-to-vertical impulse response of the equalizer. See equation (2.22).

$s_{0bh}(t)$ the input horizontal complex baseband signal of the horizontal channel. See equation (2.10).

$s_{0bv}(t)$ the input horizontal complex baseband signal of the vertical channel. See equation (2.9).

$s_{ih}(t)$ the in-phase input signal of the horizontal channel. See Fig. 2.1.

$s_{iv}(t)$ the in-phase input signal of the vertical channel. See Fig. 2.1.

$s_{Lbh}(t)$ the data transmitted in the horizontal channel after the transmitter filter. See equation (2.18).

$s_{Lbv}(t)$ the data transmitted in the vertical channel after the transmitter filter. See equation (2.18).

$s_{ph}(t)$ the modulated signal after the modulator in the horizontal channel. See equation (2.2) and Fig. 2.1.

$s_{pv}(t)$ the modulated signal after the modulator in the vertical channel. See equation (2.2) and Fig. 2.1.

$s_{qh}(t)$ the quadrature input signal of the horizontal channel. See Fig. 2.1.

$s_{qv}(t)$ the quadrature input signal of the vertical channel. See Fig. 2.2.

t the dummy parameter used as time index.

T the symbol period. See equation (2.15).

$\boldsymbol{u}(t)$ the output matrix of the channel. See equation (2.5).

$u_h(t)$ the output signal in the horizontal channel. See equation (2.5).

$u_v(t)$ the output signal in the vertical channel. See equation (2.5).

$w(t)$ the receiver bandlimiting filter impulse response. See equation (2.8).

W_c the real carrier spacing. See equation (4.5).

W_r the real receiver bandwidth. See equation (4.3).

W_t the real transmitter bandwidth. See equation (4.1).

$x_{ih}(n)$ the in-phase signal in the horizontal polarization channel. See Fig. 2.1.

$x_{iv}(n)$ the in-phase signal in the vertical polarization channel. See Fig. 2.1.

$x_{qh}(n)$ the quadrature signal in the horizontal polarization channel. See Fig. 2.1.

$x_{qv}(n)$ the quadrature signal in the vertical polarization channel. See Fig. 2.1.

$y_{bh}(t)$ the output complex baseband signal of the horizontal channel. See equation (2.12).

$y_{bv}(t)$ the output complex baseband signal of the vertical channel. See equation (2.12).

$\boldsymbol{y}_h(t)$ the output signal matrix of the horizontal channel. See equation (2.7).

$y_{ih}(t)$ the output in-phase signal in the horizontal channel. See equation (2.8).

$y_{iv}(t)$ the output in-phase signal in the vertical channel. See equation (2.8).

$y_{qh}(t)$ the output quadrature signal in the horizontal channel. See equation (2.8).

$y_{qv}(t)$ the output quadrature signal in the vertical channel. See equation (2.8).

$\boldsymbol{y}_v(t)$ the output signal matrix of the vertical channel. See equation (2.7).

$\boldsymbol{\Phi}_0(f)$ the combined channel frequency response matrix. See equation (2.23).

$\boldsymbol{\varphi}_0(t)$ the combined channel impulse response matrix for the data transmitted. See equation (2.19).

$\Phi_{0hh}(f)$ the horizontal-to-horizontal combined channel frequency response. See equation (2.23).

$\varphi_{0hh}(t)$ the baseband combined channel impulse response of the horizontal-to-vertical channel. See equation (2.19)

$\Phi_{0hv}(f)$ the horizontal-to-vertical combined channel frequency response. See equation (2.23).

$\varphi_{0hv}(t)$ the baseband combined channel impulse response of the horizontal-to-vertical channel. See equation (2.19).

$\Phi_{0vh}(f)$ the vertical-to-horizontal combined channel frequency response. See equation (2.23).

$\varphi_{0vh}(t)$ the baseband combined channel impulse response of the vertical-to-horizontal channel. See equation (2.19).

$\Phi_{0vv}(f)$ the vertical-to-vertical combined channel frequency response. See equation (2.23).

$\varphi_{0vv}(t)$ the baseband combined channel impulse response of the vertical-to-vertical channel. See equation (2.19).

$\boldsymbol{\Phi}_L(f)$ the combined co-channel frequency response matrix. See equation (2.24).

$\Phi_{Lhh}(f)$ the horizontal-to-horizontal combined co-channel frequency response. See equation (2.24).

$\varphi_{Lhh}(t)$ the L^{th} combined co-channel impulse response of horizontal-to-horizontal channel. See equation (2.21).

$\Phi_{Lhv}(f)$ the horizontal-to-vertical combined co-channel frequency response. See equation (2.24).

$\varphi_{Lhv}(t)$ the L^{th} combined co-channel impulse response of horizontal-to-vertical channel. See equation (2.21)

$\Phi_{Lvh}(f)$ the vertical-to-horizontal combined co-channel frequency response. See equation (2.24)

$\varphi_{Lvh}(t)$ the L^{th} combined co-channel impulse response of vertical-to-horizontal channel. See equation (2.21).

$\Phi_{Lvv}(f)$ the vertical-to-vertical combined co-channel frequency response. See equation (2.24).

$\varphi_{Lvv}(t)$ the L^{th} combined co-channel impulse response of vertical-to-vertical channel. See equation (2.21).

π the ratio of the circumference of a circle to its diameter.

σ^2 the variance of noise signal.

$\delta(t-nT)$ the Dirac delta function. See equation (2.15).

$*$ the matrix convolution. See (3.1).

List of Abbreviations and Acronyms

ACI	Adjacent-channel Interference
CCI	Co-channel Interference
CPI	Cross-polarization Interference
FDMA	Frequency Division Multiple Access
GZFE	Generalized Zero-Forcing Equalizer
ISI	Intersymbol Interference
MMSE	Minimum Mean Square Error
PAM	Pulse Amplitude Modulation
QAM	Quadrature Amplitude Modulation
TIR	Transmitter Impulse Response
TDMA	Time Division Multiple Access
WDMA	Wavelength Division Multiple Access

Chapter 1

Introduction

1.1 Introduction to Research

The purpose of this thesis is to obtain research results toward effective methods for combating the deleterious effects of various impairments arising in digital data transmission over dually polarized communication channels. The demand for services requires more efficient use of the spectrum. The techniques that have been employed are Frequency Division Multiple Access (FDMA), Time Division Multiple Access (TDMA) and Wavelength Division Multiple Access (WDMA).

The method of subdividing the available channel bandwidth into a number of non-overlapping sub-channels and to assign a sub-channel to each user upon request by the users is called Frequency Division Multiple Access. The method of subdividing a time frame into a number of non-overlapping subintervals then assigning to each user to access is called Time Division Multiple Access [Couch97, Lee94, Proakis01]. Wavelength Division Multiple Access utilizes the same technique as in FDMA. Instead of using frequency, the wavelength is used for multiple access in the optical domain.

These improvements in spectral efficiency can cause more co-channel interference and adjacent-channel interference. CCI is the interference between signals with similar carrier frequencies, whereas ACI is the interference between signals with different carrier frequencies that are close enough to cause overlaps in the spectrum. In wireless communications, a multipath channel can also cause CCI among cells.

Moreover, transmission of M-state Quadrature Amplitude Modulated (QAM) signals via orthogonally (vertical and horizontal) polarized carriers is an effective method for reusing existing bandwidth with the advantages of more system capacity and higher bit rate. However, the obstacle in the way of realizing these advantages is the unavoidable presence of cross-polarization interference (CPI) between two polarized signals. This impairment is due to imperfect waveguides. Besides CCI, ACI and CPI, there will be other impairments. However, the only one that is considered is additive white channel noise.

The purpose of this thesis is to obtain a fundamental understanding and derive a solution for the aforementioned problems using channel equalization and wide relative transmitter bandwidth techniques. The solution is applicable to fiber-optics, wireless and wired systems.

Channel equalization is a well-established theory when dealing with optimal communications system design [Lucky68]. It synthesizes filters related to the inverse of the distortions in a channel frequency response in order that data can be extracted from an equalized signal. It is an effective way to mitigate the intersymbol interference (ISI) and channel noise in the transmission of digital signals. Channel equalization also leads to computationally efficient implementations.

The utilization of transmitter bandwidths which are wide relative to the symbol rate technique to suppress interference has been the field of relatively little previous research compared to channel equalization. The existence of such equalization using this technique and the conditions to suppress all the interferences will be studied in detail in this thesis.

1.2 Literature Survey

1.2.1 Overview

This thesis generally deals with a fiber-optic communication system. The impairments existing in the system are typically ISI, CCI, ACI and CPI along with channel white noise.

Various researches have explored aforementioned problems [Smith68, Harri69, Kave85 and Khors99]. None has been performed for all five impairments with respect to the conditions for suppressing all interference.

1.2.2 Channel Equalization

Published research has exhibited the ability of suppressing various types of interference under certain conditions. [Lind95, Petersen90].

Based on previous work [Amitay84], Kavehrad and Salz proposed a theoretical frame work to mitigate CPI and ISI along with additive noise using finite-tap transversal structures over dually polarized multipath radio channels [Kave85]. However, CCI and ACI suppression were not considered.

1.2.3 Utilization of Wide Relative Transmitter Bandwidth

The effect of transmitter pulse bandwidth relative to the symbol rate is well known where only ISI was presented [Nyquist28]. The result of his work was that the frequency spectrum in the range from $-1/(2T)$ to $1/(2T)$ must be flat to obtain zero ISI where T is the symbol period and $1/T$ is symbol rate.

The benefit of wide relative transmitter bandwidth was foretold by Shnidman to suppress ISI and CCI where less than 100 percent excess bandwidth of $1/T$ were employed [Shnidman67].

Filters with excess bandwidth over 100 percent were described by Lind et al. for the raised cosine class of communication filters that satisfy the zero interference condition [Lind95]. It did not explore the conditions of CPI and multi user systems.

Since ISI, CCI and ACI are fundamentally equivalent signals [Gardner75, Petersen92]; this leads to the possibility that the wide transmitter bandwidth theory applicable to ISI might be applied to CCI and ACI as well.

The ability to suppress interference utilizing the benefit of wide transmitter bandwidth has been described, where over 100 percent excess bandwidth was employed [Venkatara90, Golden90, and Petersen90]. More recent development of the theory was generalized in the research of [Golden94].

Based on the previous research, this thesis will extend into the analyses of the suppression for mentioned interferences including CPI using wide transmitter bandwidth and the existence and conditions of such equalization, claimed to be the contributions of this research.

1.3 Thesis Contributions

This thesis proposes a design possibility to suppress ISI, CCI, ACI and CPI for an optical fiber communication system where two 4-QAM signals are transmitted separately over vertical and horizontal polarization channels.

The passband and baseband single user and multi user system models are illustrated. The contributions described below are interference analyses and suppression techniques for a two-polarization generalized equalizer operating in the presence of ISI, CCI, ACI, CPI and additive white noise.

The ability to suppress ISI, CCI and CPI by a generalized zero-forcing equalizer is presented and the conditions to suppress ISI, CCI and CPI are also explained.

The first result deals with the ability to suppress ISI, CCI and CPI by a generalized zero-forcing equalizer. It is found that such equalization does exist in ISI, CCI and CPI.

The second result is the analyses of the conditions for the existence of a generalized zero-forcing equalizer in ISI, CCI and CPI. Every increase in transmitter bandwidth, equal to the symbol rate, provides the ability to completely suppress an additional interferer by means of linear equalization.

The third result is the discovery of the ability of the generalized zero-forcing equalizer to suppress ACI in addition to ISI, CCI and CPI. The expressions of determining number of interferers, number of unknowns and number of equations are given.

The fourth contribution deals with the conditions of such equalization which incorporates the effect of transmitter bandwidth, carrier spacing and number of ACI interferers. Under certain conditions, the increase in receiver bandwidth may have the ability to assist the equalizer to suppress more ACI interferers.

1.4 Thesis Outline

In Chapter 1, the motivation, the background and the literature survey are described. This section outlines how this thesis is organized to make those contributions.

In Chapter 2, the passband and baseband models for both single user and multi-user systems including transmitter, channel and the receiver are presented. Four types of interferences are introduced. System impulse responses including transmitter impulse response, channel impulse response, receiver impulse response, combined channel impulse response, equalized combined impulse response are defined. The system is assumed to transmit the data of interest using the rate which is identical to that of the interferers and all the data are mutually uncorrelated.

Chapter 3 shows the analyses of the generalized zero-forcing equalizer in the presence of ISI, CCI and CPI. First, zero interference conditions for the aforementioned interference are defined in both time and frequency domain. The mathematical expressions of equalized combined channel impulse response and co-channel impulse response are described. With an increase in excess bandwidth, the number of co-channel interferers and the ability to suppress ISI, CCI and CPI are analyzed. The results show the existence of such an equalizer and every increase in total bandwidth equal to the symbol rate can suppress an additional interferer.

Chapter 4 extends the analyses of interference suppression when ACI is present in the system. The transmitter, carrier and receiver bandwidths are examined followed by how the equalized combined channel impulse response and the equalized combined co-channel impulse response are constructed. There are two situations considered for the equalizer analyses. One is the non-overlapping case; the other is the overlapping case. It is discovered that such an equalizer has the ability to suppress ACI in addition to ISI, CCI and CPI. Also, it is found that an increase in receiver bandwidth may have the ability to assist the equalizer to suppress more ACI interferers. The expressions determining number of interferers, the number of unknowns and number of equations are given.

In Chapter 5, the thesis is concluded and the discussion of the future work is also described.

Chapter 2

System Model

2.1 Passband System Model

Fig. 2.1 is a block diagram of the passband single-user system model.

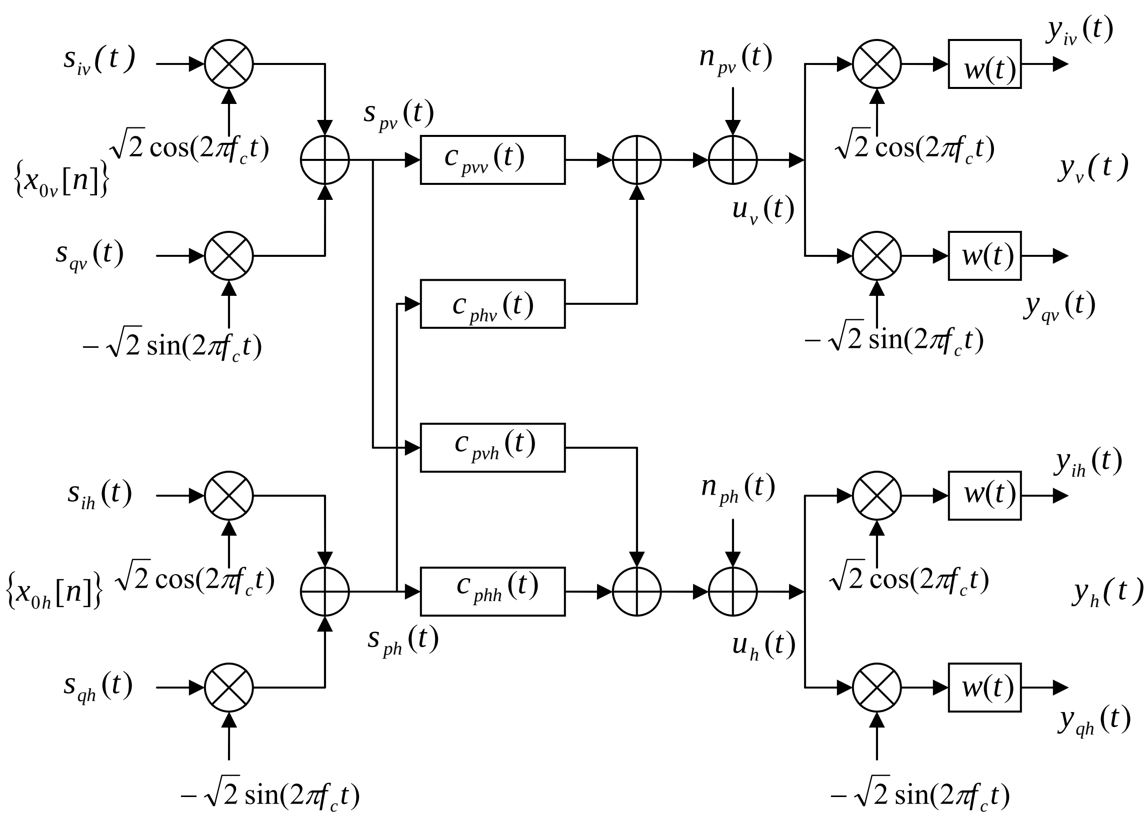


Fig. 2.1 Passband Single-User System Model

The communication is digital. The data is transmitted using a linear modulation scheme. Two independent four-state quadrature amplitude modulation (QAM) signals, converted from pulse amplitude modulation (PAM) signals, are used in this thesis. One digital QAM signal, $x_{0v}(n)$, the input signal, is transmitted over a vertical channel and the other input signal, $x_{0h}(n)$, is transmitted over a horizontal channel,

$$x_{0v}(n) = \begin{bmatrix} x_{iv}(n) \\ x_{qv}(n) \end{bmatrix}, \; x_{0h}(n) = \begin{bmatrix} x_{ih}(n) \\ x_{qh}(n) \end{bmatrix}, \qquad (2.1)$$

where $x_{iv}(n)$ is the in-phase signal in the vertical polarization channel,

$x_{qv}(n)$ is the quadrature signal in the vertical polarization channel,

$x_{ih}(n)$ is the in-phase signal in the horizontal polarization channel,

$x_{qh}(n)$ is the quadrature signal in the horizontal polarization channel, and

f_c denotes the carrier frequency.

All the data transmitted are using T as the symbol period, and $1/T$ is the symbol rate. The digital modulation is described further in section 2.3.

The above signals are then passed to modulators. The modulated signals are $s_{pv}(t)$ and $s_{ph}(t)$,

$$s_{pv}(t) = \sqrt{2}\left[s_{iv}(t)\cos(2\pi f_c t) - s_{qv}(t)\sin(2\pi f_c t)\right],$$

$$s_{ph}(t) = \sqrt{2}\left[s_{ih}(t)\cos(2\pi f_c t) - s_{qh}(t)\sin(2\pi f_c t)\right], \qquad (2.2)$$

where $s_{pv}(t)$ is the modulated signal after the modulator in the vertical channel, and

$s_{ph}(t)$ is the modulated signal after the modulator in the horizontal channel.

The fiber optic channel is characterized by a passband channel impulse response matrix, $\boldsymbol{c}_p(t)$,

$$\boldsymbol{c}_p(t) = \begin{bmatrix} c_{pvv}(t) & c_{pvh}(t) \\ c_{phv}(t) & c_{phh}(t) \end{bmatrix}, \qquad (2.3)$$

where $c_{pvv}(t)$ is the passband vertical-to-vertical channel impulse response,

$c_{pvh}(t)$ is the passband vertical-to-horizontal channel impulse response,

$c_{phv}(t)$ is the passband horizontal-to-vertical channel impulse response, and

$c_{phh}(t)$ is the passband horizontal-to-horizontal channel impulse response.

The passband channel impulse responses are related to the baseband channel impulse responses by the following equations,

$$c_{pvv}(t) = 2\,\mathrm{Re}\left\{c_{bvv}(t)e^{j2\pi f_c t}\right\},$$

$$c_{pvh}(t) = 2\,\mathrm{Re}\left\{c_{bvh}(t)e^{j2\pi f_c t}\right\},$$

$$c_{phv}(t) = 2\,\mathrm{Re}\left\{c_{bhv}(t)e^{j2\pi f_c t}\right\}, \text{ and}$$

$$c_{phh}(t) = 2\,\mathrm{Re}\left\{c_{bhh}(t)e^{j2\pi f_c t}\right\}, \qquad (2.4)$$

where $\mathrm{Re}\{\bullet\}$ denotes the real part of $\bullet$, $c_{bvv}(t)$ is the baseband vertical-to-vertical channel impulse response, $c_{bvh}(t)$ is the baseband vertical-to-horizontal channel impulse response, $c_{bhv}(t)$ is the baseband horizontal-to-vertical channel impulse response, and $c_{bhh}(t)$ is the baseband horizontal-to-horizontal channel impulse response.

The output of the channel, $\boldsymbol{u}(t)$, [Kave85, Lu00], is

$$\boldsymbol{u}(t) = \begin{bmatrix} u_v(t) \\ u_h(t) \end{bmatrix} = \begin{bmatrix} s_{pv}(t) * c_{pvv}(t) + s_{ph}(t) * c_{phv}(t) \\ s_{ph}(t) * c_{phh}(t) + s_{pv}(t) * c_{pvh}(t) \end{bmatrix} + \boldsymbol{n}_p(t), \tag{2.5}$$

where $*$ denotes convolution,

$u_v(t)$ is the output signal in the vertical channel,

$u_h(t)$ is the output signal in the horizontal channel, and

$\boldsymbol{n}_p(t)$ is the channel white noise vector with the components in both vertical and horizontal channels, $n_{pv}(t)$ and $n_{ph}(t)$,

$$\boldsymbol{n}_p(t) = \begin{bmatrix} n_{pv}(t) \\ n_{ph}(t) \end{bmatrix}. \tag{2.6}$$

The noise signals, $n_{pv}(t)$ and $n_{ph}(t)$, are zero mean and uncorrelated with variance of σ^2.

At the receiver end, $\boldsymbol{u}(t)$ is demodulated before passing through receiver bandlimiting filters $w(t)$. The output signals to the receivers are:

$$\boldsymbol{y}_v(t) = \begin{bmatrix} y_{iv}(t) \\ y_{qv}(t) \end{bmatrix},$$

$$\boldsymbol{y}_h(t) = \begin{bmatrix} y_{ih}(t) \\ y_{qh}(t) \end{bmatrix}, \tag{2.7}$$

and,

$$y_{iv}(t) = \sqrt{2}\left[u_v(t)\cos(2\pi f_c t)\right] * w(t),$$

$$y_{qv}(t) = -\sqrt{2}\left[u_v(t)\sin(2\pi f_c t)\right] * w(t),$$

$$y_{ih}(t) = \sqrt{2}\left[u_h(t)\cos(2\pi f_c t)\right] * w(t),$$

$$y_{qh}(t) = -\sqrt{2}\left[u_h(t)\sin(2\pi f_c t)\right] * w(t), \tag{2.8}$$

where $y_{iv}(t)$ is the output in-phase signal in the vertical channel,

$y_{qv}(t)$ is the output quadrature signal in the vertical channel,

$y_{ih}(t)$ is the output in-phase signal in the horizontal channel,

$y_{qh}(t)$ is the output quadrature signal in the horizontal channel, and

$w(t)$ is the receiver bandlimiting filter impulse response. It is used to cut off the high frequency component, $2f_c$, and preserve the desired component, f_c.

The received signals, $y_v(t)$ and $y_h(t)$, will then pass through a linear equalizer.

This concludes the description of passband single-user system model.

2.2 Baseband System Model

Based on the passband single-user system model described in the previous section, a baseband single-user model can be constructed.

Fig. 2.2 is a block diagram of a baseband single-user system model. This model, also known as the low-pass equivalent model, utilizes the complex envelope of a passband signal [Lee94]. It is commonly used in simulation of a passband signal to reduce the computation load.

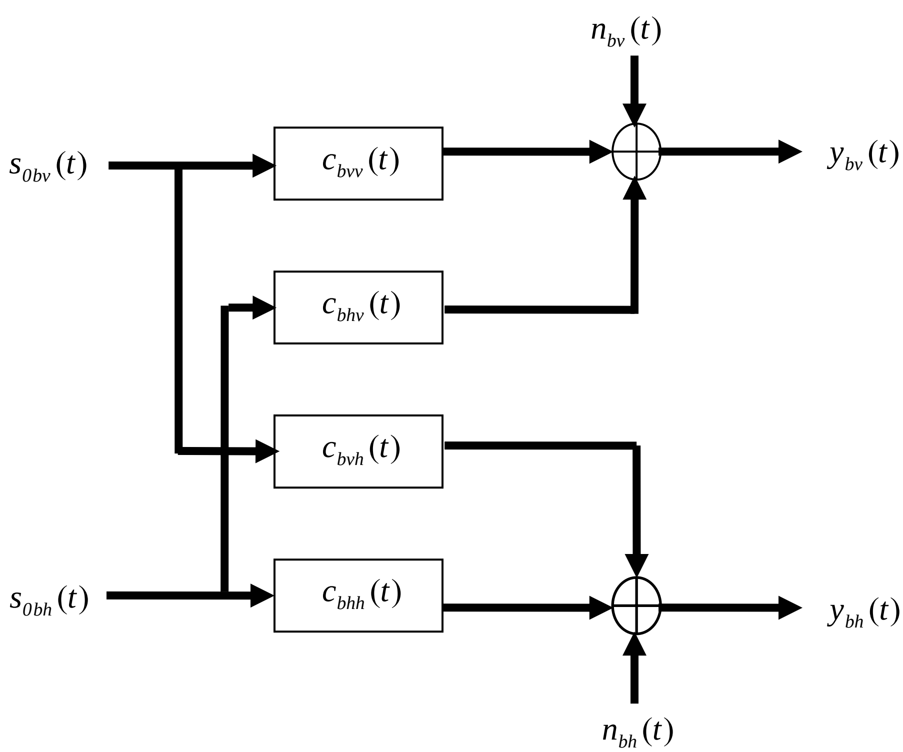


Fig. 2.2 Baseband Single-User System Model

From Figs. 2.1 and 2.2, we have

$$s_{pv}(t) = \sqrt{2}\left[s_{iv}(t)\cos(2\pi f_c t) - s_{qv}(t)\sin(2\pi f_c t)\right]$$

$$= \sqrt{2}\,\mathrm{Re}\left\{ \left[s_{iv}(t) + js_{qv}(t)\right] \; e^{j2\pi f_c t} \right\}$$

$$= \sqrt{2}\,\mathrm{Re}\left\{s_{bv}(t)e^{j2\pi f_c t}\right\}$$

Therefore, the relation between the vertical complex baseband waveform $s_{0bv}(t)$ and two passband input signals, $s_{iv}(t)$ and $s_{qv}(t)$, is obtained,

$$s_{0bv}(t) = s_{iv}(t) + js_{qv}(t)\,. \qquad (2.9)$$

Similarly, we can also obtain the expression for the horizontal complex baseband waveform, $s_{0bh}(t)$,

$$s_{0bh}(t) = s_{ih}(t) + js_{qh}(t)\,. \qquad (2.10)$$

The corresponding expressions for output complex baseband waveform $y_{bv}(t)$ in the vertical channel and $y_{bh}(t)$ in the horizontal channel are:

$$y_{bv}(t) = y_{iv}(t) + jy_{qv}(t)\text{, and} \qquad (2.11)$$

$$y_{bh}(t) = y_{ih}(t) + jy_{qh}(t)\,. \qquad (2.12)$$

The complex baseband channel impulse response matrix becomes:

$$\boldsymbol{c}_b(t) = \begin{bmatrix} c_{bvv}(t) & c_{bvh}(t) \\ c_{bhv}(t) & c_{bhh}(t) \end{bmatrix}, \qquad (2.13)$$

where $c_{bvv}(t)$ is baseband vertical-to-vertical channel impulse response,

$c_{bvh}(t)$ is the baseband vertical-to-horizontal channel impulse response,

$c_{bhv}(t)$ is the baseband horizontal-to-vertical channel impulse response, and

$c_{bhh}(t)$ is the baseband horizontal-to-horizontal channel impulse response.

$n_{bv}(t)$ and $n_{bh}(t)$ are the corresponding components of the noise vector, $\boldsymbol{n}_b(t)$, in the vertical and horizontal channels,

$$\boldsymbol{n}_b(t) = \begin{bmatrix} n_{bv}(t) \\ n_{bh}(t) \end{bmatrix}. \tag{2.14}$$

2.3 Baseband Multi-user System Model

Fig. 2.3 illustrates the digital inputs of the system.

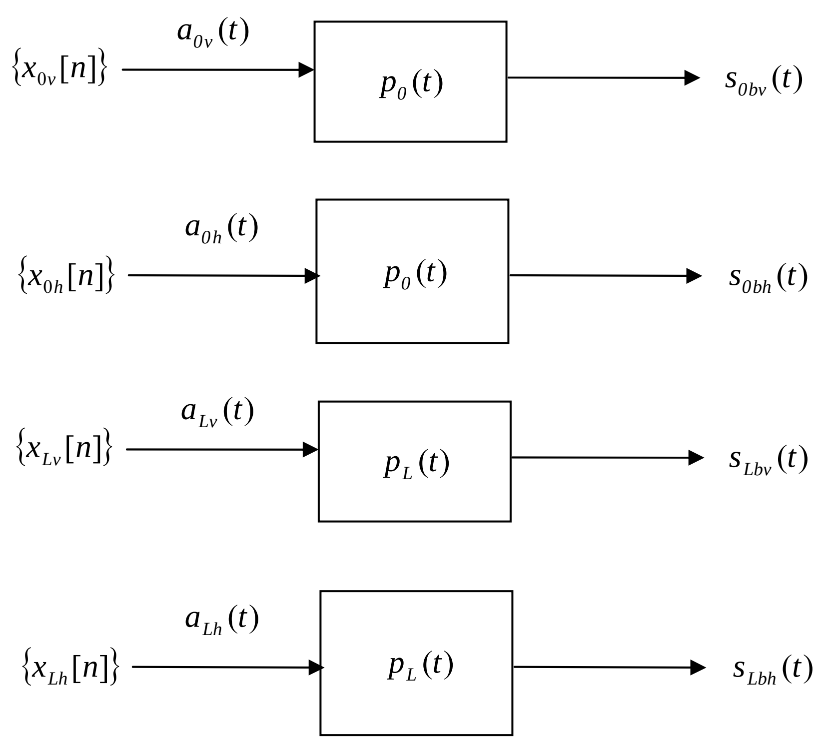


Fig. 2.3 Input Signals of the System

In Fig. 2.3, the notation $\{x_{0v}[n]\}$ and $\{x_{0h}[n]\}$ applied to the input of the transmitter filters mean that the data is modulated upon a sequence of Dirac delta functions so that the inputs to the transmitter filters, $p_0(t)$, for the data transmitted, are the following signals:

$$a_{ov}(t) = \sum_{n=-\infty}^{+\infty} x_{0v}[n]\,\delta(t - nT)$$

$$a_{oh}(t) = \sum_{n=-\infty}^{+\infty} x_{0h}[n]\,\delta(t - nT)\,, \qquad (2.15)$$

where $\delta(t - nT)$ denotes Dirac delta function.

This means that the outputs of the transmitters are the signals which are linearly modulated with the data:

$$s_{0bv}(t) = \sum_{n=-\infty}^{+\infty} x_{0v}[n]\; p_0(t - nT)$$

$$s_{0bh}(t) = \sum_{n=-\infty}^{+\infty} x_{0h}[n]\; p_0(t - nT) \qquad (2.16)$$

where

$p_0(t)$ is the transmitter impulse response for the data transmitted,

$s_{0bv}(t)$ is the data transmitted in the vertical channel after the transmitter filter, and

$s_{0bh}(t)$ is the data transmitted in the horizontal channel after the transmitter filter.

Similarly, the inputs of the other co-channels can be defined in the same way:

$$a_{Lv}(t) = \sum_{n=-\infty}^{+\infty} x_{Lv}[n]\, \delta(t - nT)$$

$$a_{Lh}(t) = \sum_{n=-\infty}^{+\infty} x_{Lh}[n]\, \delta(t - nT) \qquad (2.17)$$

where

$\{x_{Lv}[n]\}$ and $\{x_{Lh}[n]\}$ denote the inputs of the L th co-channel,

$a_{Lv}(t)$ and $a_{Lh}(t)$ are the inputs of the transmitter filter, $p_L(t)$, for the L th co-channel in the vertical and horizontal channel, respectively, and

L is from a set of integers, $\{1,2,3,...\}$.

The outputs of the transmitter for the L th co-channel are:

$$s_{Lbv}(t) = \sum_{n=-\infty}^{+\infty} x_{Lv}[n]\; p_L(t - nT)$$

$$s_{Lbh}(t) = \sum_{n=-\infty}^{+\infty} x_{Lh}[n]\; p_L(t - nT), \qquad (2.18)$$

where

$p_L(t)$ is the transmitter impulse response for the L th co-channel,

$s_{Lbv}(t)$ is the data transmitted in the vertical channel after the transmitter filter, and

$s_{Lbh}(t)$ is the data transmitted in the horizontal channel after the transmitter filter.

Fig. 2.4 is a block diagram of baseband multi-user system model.

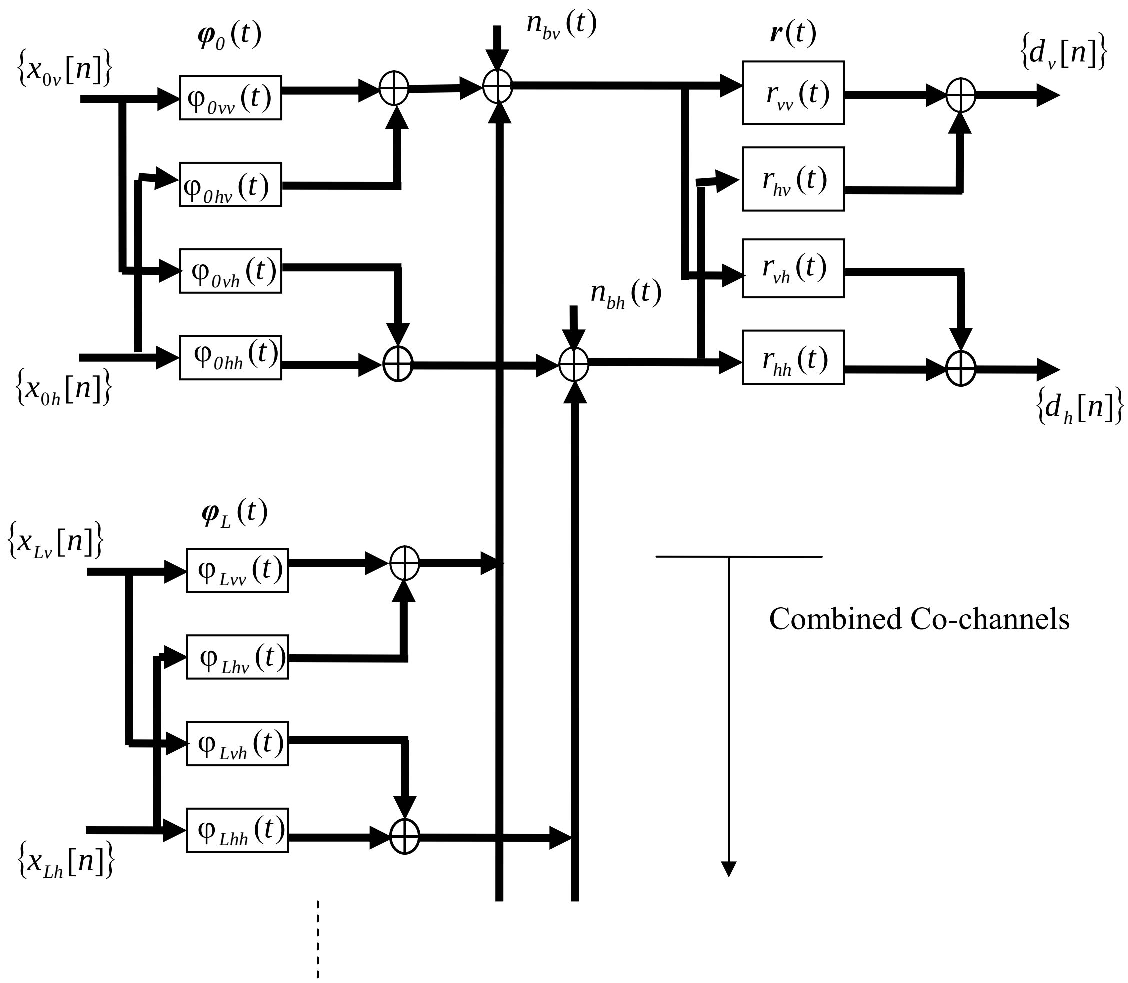


Fig. 2.4 Baseband Multi-User System Model

The data transmitted are $x_{0v}(n)$ in the vertical channel, and $x_{0h}(n)$, in the horizontal channel. Other co-channels are represented by $x_{Lv}(n)$ in the vertical channel, and $x_{Lh}(n)$ in the horizontal channel. The data transmitted in the co-channels are statistically independent and use the same symbol rate, $1/T$, as that used to transmit the data.

Define the combined channel impulse response matrix for the data transmitted as $\boldsymbol{\varphi}_0(t)$,

$$\boldsymbol{\varphi}_0(t) = \begin{bmatrix} \varphi_{0vv}(t) & \varphi_{0vh}(t) \\ \varphi_{0hv}(t) & \varphi_{0hh}(t) \end{bmatrix} \tag{2.19}$$

where

$\varphi_{0vv}(t)$ is the baseband combined channel impulse response of the vertical-to-vertical channel,

$\varphi_{0vh}(t)$ is the baseband combined channel impulse response of the vertical-to-horizontal channel,

$\varphi_{0hv}(t)$ is the baseband combined channel impulse response of the horizontal-to-vertical channel, and

$\varphi_{0hh}(t)$ is the baseband combined channel impulse response of the horizontal-to-vertical channel. The first subscript "0" denotes the channel that transmits the data of interest. The second and the third subscripts are either "v" or "h", denoting "vertical" or "horizontal", respectively. The second subscript denotes the input and the third subscript denotes the output.

The mathematical expressions of $\varphi_{0vv}(t)$, $\varphi_{0vh}(t)$, $\varphi_{0hv}(t)$ and $\varphi_{0hh}(t)$ are:

$$\varphi_{0vv}(t) = p_0(t) * c_{bvv}(t)$$

$$\varphi_{0vh}(t) = p_0(t) * c_{bvh}(t)$$

$$\varphi_{0hv}(t) = p_0(t) * c_{bhv}(t)$$

$$\varphi_{0hh}(t) = p_0(t) * c_{bhh}(t) \qquad (2.20)$$

Fig. 2.5 is an illustration of how the combined channel impulse response matrix $\boldsymbol{\varphi}_0(t)$ is formed.

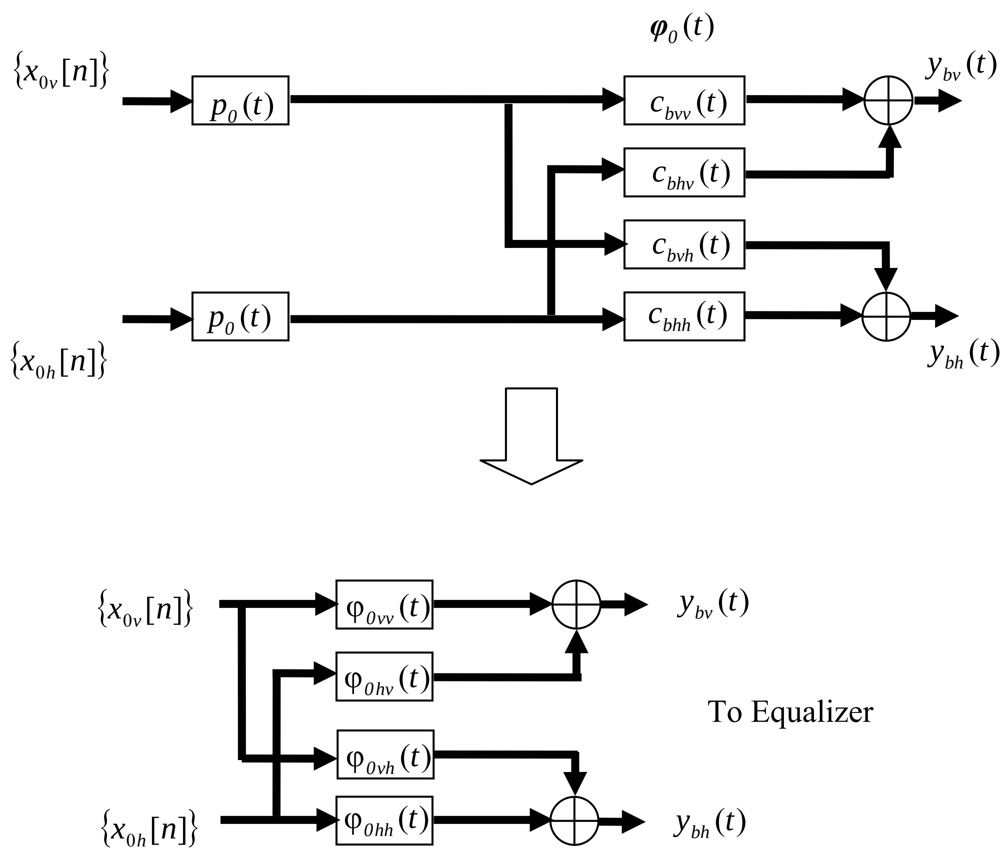


Fig. 2.5 Definition of Combined Channel Impulse Response

Based on equation (2.20), the combined co-channel impulse response matrix as $\boldsymbol{\varphi}_L(t)$, can be formed in the same way. Define the combined co-channel impulse response matrix as $\boldsymbol{\varphi}_L(t)$,

$$\boldsymbol{\varphi}_L(t) = \begin{bmatrix} \varphi_{Lvv}(t) & \varphi_{Lvh}(t) \\ \varphi_{Lhv}(t) & \varphi_{Lhh}(t) \end{bmatrix}, \tag{2.21}$$

where

$\varphi_{Lvv}(t)$ is the Lth combined co-channel impulse response of vertical-to-vertical channel,

$\varphi_{Lvh}(t)$ is the Lth combined co-channel impulse response of vertical-to-horizontal channel,

$\varphi_{Lhv}(t)$ is the Lth combined co-channel impulse response of horizontal-to-vertical channel, and

$\varphi_{Lhh}(t)$ is the Lth combined co-channel impulse response of horizontal-to-horizontal channel.

The output of the channel will pass through a linear equalizer.

Define the impulse response matrix of the equalizer as $\boldsymbol{r}(t)$,

$$\boldsymbol{r}(t) = \begin{bmatrix} r_{vv}(t) & r_{vh}(t) \\ r_{hv}(t) & r_{hh}(t) \end{bmatrix} \tag{2.22}$$

where

$r_{vv}(t)$ is the vertical-to-vertical impulse response of the equalizer,

$r_{vh}(t)$ is the vertical-to-horizontal impulse response of the equalizer,

$r_{hv}(t)$ is the horizontal-to-vertical impulse response of the equalizer, and

$r_{hh}(t)$ is the horizontal-to-horizontal impulse response of the equalizer.

The equalizer is considered have impulse responses which are time invariant meaning that adaptive equalizers can be used for implementation.

The outputs of the equalizer are $d_v[n]$ and $d_h[n]$. They are fed to a series of samplers and decision devices for quantizing. The outputs of the quantizer will be the estimates of the data transmitted.

2.4 System Frequency Response

According to equation (2.19), define the combined channel frequency response, $\boldsymbol{\Phi}_0(f)$, for the data transmitted is the Fourier transform of its impulse response, $\boldsymbol{\varphi}_0(t)$,

$$\boldsymbol{\Phi}_0(f) = \begin{bmatrix} \Phi_{0vv}(f) & \Phi_{0vh}(f) \\ \Phi_{0hv}(f) & \Phi_{0hh}(f) \end{bmatrix} \qquad (2.23)$$

where

$\Phi_{0vv}(f)$ is the vertical-to-vertical combined channel frequency response,

$\Phi_{0vh}(f)$ is the vertical-to-horizontal combined channel frequency response,

$\Phi_{0hv}(f)$ is the horizontal-to-vertical combined channel frequency response, and

$\Phi_{0hh}(f)$ is the horizontal-to-horizontal combined channel frequency response.

Since the impulse response of the co-channel completely characterized by its transmitter impulse response, the channel impulse response and receiver impulse response, we can easily conclude that the combined co-channel impulse response is constructed in the similar fashion.

According to equation (2.21), define the frequency response of combined co-channel, $\boldsymbol{\Phi}_0(f)$, as the Fourier transform of its impulse response, $\boldsymbol{\varphi}_L(t)$,

$$\boldsymbol{\Phi}_L(f) = \begin{bmatrix} \Phi_{Lvv}(f) & \Phi_{Lvh}(f) \\ \Phi_{Lhv}(f) & \Phi_{Lhh}(f) \end{bmatrix} \tag{2.24}$$

where

$\Phi_{Lvv}(f)$ is the vertical-to-vertical combined co-channel frequency response,

$\Phi_{Lvh}(f)$ is the vertical-to-horizontal combined co-channel frequency response,

$\Phi_{Lhv}(f)$ is the horizontal-to-vertical combined co-channel frequency response,

and

$\Phi_{Lhh}(f)$ is the horizontal-to-horizontal combined co-channel frequency response.

Define the frequency response of the equalizer, $\boldsymbol{R}(f)$, as the Fourier transform of its impulse response, $\boldsymbol{r}(t)$,

$$\boldsymbol{R}(f) = \begin{bmatrix} R_{vv}(f) & R_{vh}(f) \\ R_{hv}(f) & R_{hh}(f) \end{bmatrix} \tag{2.25}$$

where

$R_{vv}(f)$ is the receiver vertical-to-vertical channel frequency response,

$R_{vh}(f)$ is the receiver vertical-to-horizontal channel frequency response,

$R_{hv}(f)$ is the receiver horizontal-to-vertical channel frequency response, and

$R_{hh}(f)$ is the receiver horizontal-to-horizontal channel frequency response.

2.5 Bandwidth

Define total bandwidth to be the difference between the maximum frequency and the minimum frequency. Define bandwidth to be the half of the total bandwidth. Fig. 2.6

illustrates these definitions [Petersen92].

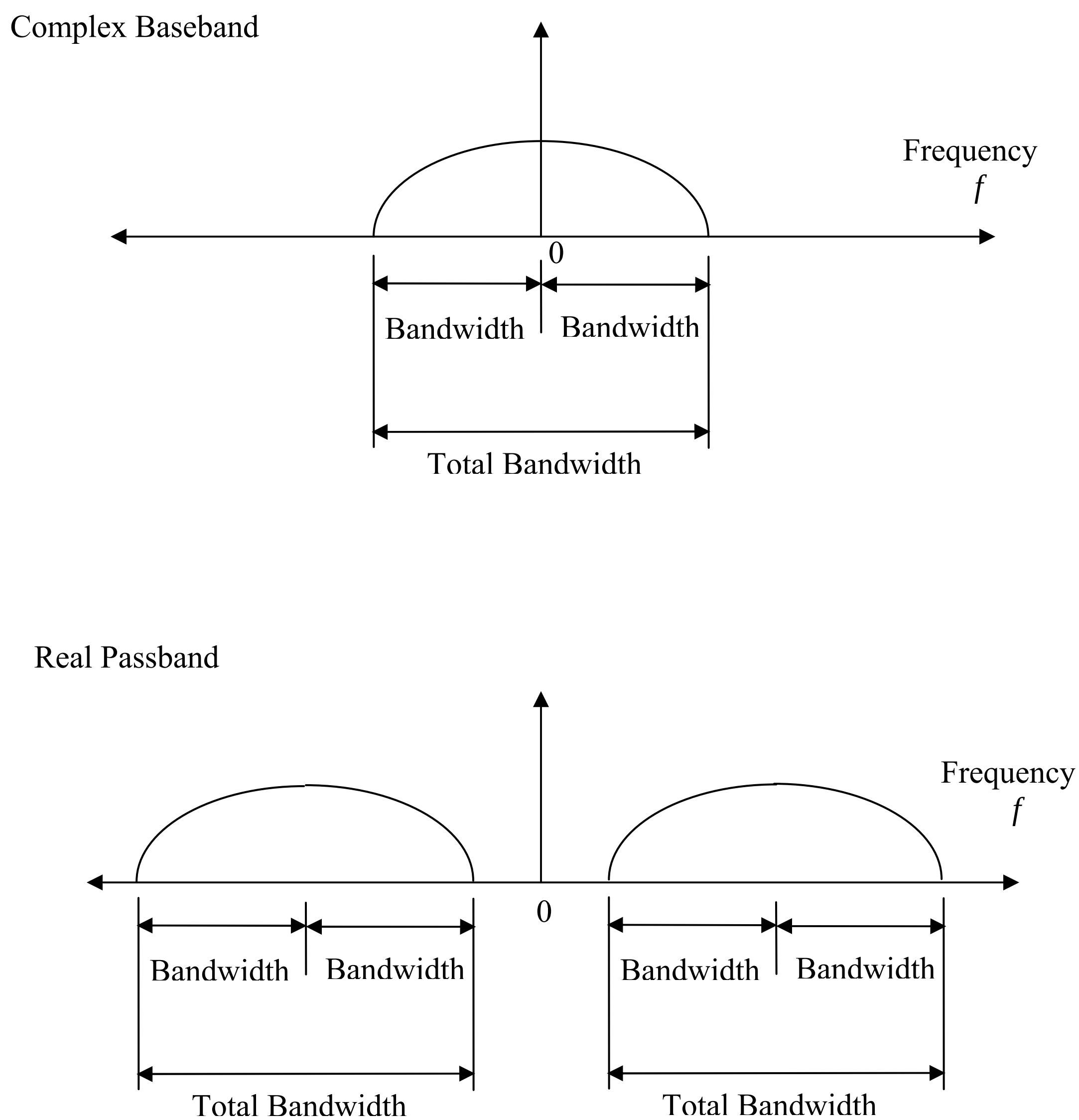


Fig. 2.6 Definitions of Bandwidth and Total Bandwidth

2.6 Interference

The interference considered in this thesis are ISI, CCI, ACI, CPI and white noise. In frequency domain, CCI is centered at 0 Hz and ACI is centered at some set of non-zero carrier frequencies. CPI is caused by the two dually polarized signals and its impulse responses are represented by $\varphi_{0vh}(t)$ and $\varphi_{0hv}(t)$ in Fig. 2.4. The white noise is represented by $n_{bv}(t)$ and $n_{bh}(t)$ with two-sided power spectral density.

Chapter 3

Equalizer Analyses for ISI, CCI and CPI

3.1 Performance Criterion

The performance criterion is the existence of a linear equalizer that has the flexibility to suppress all ISI, CCI, CPI and ACI. Such an equalizer is called a generalized zero-forcing equalizer [Shnidman67]. The analyses in this chapter are to develop a new result for ISI, CCI, and CPI which states that relatively wider bandwidths, with respect to the symbol rate, may provide the flexibility to an equalizer to suppress larger numbers of interferers. This means that this technique does not guarantee improved suppression capability under all conditions. However, under certain conditions, wide relative bandwidths allow for substantial equalizer performance improvements over the case where narrow relative bandwidths are used.

3.2 Equalized Combined Channel and Co-channels

In Fig. 2.4, define the equalized combined channel impulse response for the data transmitted to be:

$$\boldsymbol{h}_0(t) = \boldsymbol{\varphi}_0(t) * \boldsymbol{r}(t) \qquad (3.1)$$

where $*$ denotes matrix convolution [Salz 85].

There are a total of L interfering co-channels in the system, where L is from the set of natural numbers, $\{1,2,3,...\}$.

Define the equalized combined co-channels impulse responses to be:

$$\boldsymbol{h}_L(t) = \boldsymbol{\varphi}_L(t) * \boldsymbol{r}(t) \tag{3.2}$$

Define the frequency response of the equalized combined channel to be its Fourier transform, therefore equation (3.1) can be written in the frequency domain as:

$$\boldsymbol{H}_0(f) = \boldsymbol{\Phi}_0(f)\boldsymbol{R}(f) \tag{3.3}$$

Define the frequency response of the equalized combined co-channel to be its Fourier transform, therefore equation (3.2) can be written in frequency domain as:

$$\boldsymbol{H}_L(f) = \boldsymbol{\Phi}_L(f)\boldsymbol{R}(f) \tag{3.4}$$

Substituting equations (2.23) and (2.25) into equation (3.3), we have:

$$\boldsymbol{H}_0(f) = \begin{bmatrix} \Phi_{0vv}(f) & \Phi_{0vh}(f) \\ \Phi_{0hv}(f) & \Phi_{0hh}(f) \end{bmatrix} \begin{bmatrix} R_{vv}(f) & R_{vh}(f) \\ R_{hv}(f) & R_{hh}(f) \end{bmatrix}$$

$$= \begin{bmatrix} \Phi_{0vv}(f)R_{vv}(f) + \Phi_{0vh}(f)R_{hv}(f) & \Phi_{0vv}(f)R_{vh}(f) + \Phi_{0vh}(f)R_{hh}(f) \\ \Phi_{0hv}(f)R_{vv}(f) + \Phi_{0hh}(f)R_{hv}(f) & \Phi_{0hv}(f)R_{vh}(f) + \Phi_{0hh}(f)R_{hh}(f) \end{bmatrix}$$

(3.5)

Let $H_{0vv}(f) = \Phi_{0vv}(f)R_{vv}(f) + \Phi_{0vh}(f)R_{hv}(f)$

$$H_{0vh}(f) = \Phi_{0vv}(f)R_{vh}(f) + \Phi_{0vh}(f)R_{hh}(f)$$

$$H_{0hv}(f) = \Phi_{0hv}(f)R_{vv}(f) + \Phi_{0hh}(f)R_{hv}(f)$$

$$H_{0hh}(f) = \Phi_{ohv}(f)R_{vh}(f) + \Phi_{0hh}(f)R_{hh}(f) \tag{3.6}$$

Then equation (3.5) can be written as:

$$\boldsymbol{H}_0(f) = \begin{bmatrix} H_{0vv}(f) & H_{0vh}(f) \\ H_{0hv}(f) & H_{0hh}(f) \end{bmatrix}$$

$$= \begin{bmatrix} \Phi_{0vv}(f)R_{vv}(f) + \Phi_{0vh}(f)R_{hv}(f) & \Phi_{0vv}(f)R_{vh}(f) + \Phi_{0vh}(f)R_{hh}(f) \\ \Phi_{0hv}(f)R_{vv}(f) + \Phi_{0hh}(f)R_{hv}(f) & \Phi_{0hv}(f)R_{vh}(f) + \Phi_{0hh}(f)R_{hh}(f) \end{bmatrix}$$

(3.7)

where $H_{0vv}(f)$ is the vertical-to-vertical channel frequency response of the equalized combined channel,

$H_{0vh}(f)$ is the vertical-to-horizontal channel frequency response of the equalized combined channel,

$H_{0hv}(f)$ is the horizontal-to-vertical frequency response of the equalized combined channel, and

$H_{0hh}(f)$ is the horizontal-to-horizontal frequency response of the equalized combined channel.

Note that $H_{0vh}(f)$ and $H_{0hv}(f)$ are the sources of CPI.

Similarly, substitute equations (2.24) and (2.25) into equation (3.4), we have:

$$\boldsymbol{H}_L(f) = \begin{bmatrix} \Phi_{Lvv}(f) & \Phi_{Lvh}(f) \\ \Phi_{Lhv}(f) & \Phi_{Lhh}(f) \end{bmatrix} \begin{bmatrix} R_{vv}(f) & R_{vh}(f) \\ R_{hv}(f) & R_{hh}(f) \end{bmatrix}$$

$$= \begin{bmatrix} \Phi_{Lvv}(f)R_{vv}(f) + \Phi_{Lvh}(f)R_{hv}(f) & \Phi_{Lvv}(f)R_{vh}(f) + \Phi_{Lvh}(f)R_{hh}(f) \\ \Phi_{Lhv}(f)R_{vv}(f) + \Phi_{Lhh}(f)R_{hv}(f) & \Phi_{Lhv}(f)R_{vh}(f) + \Phi_{Lhh}(f)R_{hh}(f) \end{bmatrix}$$

(3.8)

Let $H_{Lvv}(f) = \Phi_{Lvv}(f)R_{vv}(f) + \Phi_{Lvh}(f)R_{hv}(f)$

$$H_{Lvh}(f) = \Phi_{Lvv}(f)R_{vh}(f) + \Phi_{Lvh}(f)R_{hh}(f)$$

$$H_{Lhv}(f) = \Phi_{Lhv}(f)R_{vv}(f) + \Phi_{Lhh}(f)R_{hv}(f)$$

$$H_{Lhh}(f) = \Phi_{Lhv}(f)R_{vh}(f) + \Phi_{Lhh}(f)R_{hh}(f) \qquad (3.9)$$

Then equation (3.8) can be written as:

$$\boldsymbol{H}_L(f) = \begin{bmatrix} H_{Lvv}(f) & H_{Lvh}(f) \\ H_{Lhv}(f) & H_{Lhh}(f) \end{bmatrix}$$

$$= \begin{bmatrix} \Phi_{Lvv}(f)R_{vv}(f) + \Phi_{Lvh}(f)R_{hv}(f) & \Phi_{Lvv}(f)R_{vh}(f) + \Phi_{Lvh}(f)R_{hh}(f) \\ \Phi_{Lhv}(f)R_{vv}(f) + \Phi_{Lhh}(f)R_{hv}(f) & \Phi_{Lhv}(f)R_{vh}(f) + \Phi_{Lhh}(f)R_{hh}(f) \end{bmatrix}$$

(3.10)

where $H_{Lvv}(f)$ is the vertical-to-vertical channel frequency response of the equalized combined co-channels,

$H_{Lvh}(f)$ is the vertical-to-horizontal channel frequency response of the equalized combined co-channels,

$H_{Lhv}(f)$ is the horizontal-to-vertical frequency response of the equalized combined co-channels, and

$H_{Lhh}(f)$ is the horizontal-to-horizontal frequency response of the equalized combined co-channels, and L denotes the Lth co-channel.

3.3 Equalizer Analysis

3.3.1 Zero ISI, CPI and CCI Conditions

Consider equation (3.7), define equalized combined channel impulse response to be:

$$\boldsymbol{h}_0(t) = \begin{bmatrix} h_{0vv}(t) & h_{0vh}(t) \\ h_{0hv}(t) & h_{0hh}(t) \end{bmatrix} \qquad (3.11)$$

where $h_{0vv}(t)$, the inverse Fourier transform of $H_{0vv}(f)$, is the vertical-to-vertical channel impulse response of the equalized combined channel,

$h_{0vh}(t)$, the inverse Fourier transform of $H_{0vh}(f)$, is the vertical-to-horizontal channel impulse response of the equalized combined channel,

$h_{0hv}(t)$, the inverse Fourier transform of $H_{0hv}(f)$, is the horizontal-to-vertical impulse response of the equalized combined channel, and

$h_{0hh}(t)$, the inverse Fourier transform of $H_{0hh}(f)$, is the horizontal-to-horizontal impulse response of the equalized combined channel.

Consider equation (3.10), define the equalized combined co-channels impulse response to be:

$$\boldsymbol{h}_L(t) = \begin{bmatrix} h_{Lvv}(t) & h_{Lvh}(t) \\ h_{Lhv}(t) & h_{Lhh}(t) \end{bmatrix} \tag{3.12}$$

where $h_{Lvv}(t)$, the inverse Fourier transform of $H_{Lvv}(f)$, is the vertical-to-vertical channel impulse response of the equalized combined co-channel,

$h_{Lvh}(t)$, the inverse Fourier transform of $H_{Lvh}(f)$, is the vertical-to-horizontal channel impulse response of the equalized combined co-channel,

$h_{Lhv}(t)$, the inverse Fourier transform of $H_{Lhv}(f)$, is the horizontal-to-vertical impulse response of the equalized combined co-channel, and

$h_{Lhh}(t)$, the inverse Fourier transform of $H_{Lhh}(f)$, is the horizontal-to-horizontal impulse response of the equalized combined co-channel.

In order to achieve zero ISI and CPI, the following expressions must be true:

$$h_{0vv}(nT) = \delta[n] \tag{3.13}$$

$$h_{0hh}(nT) = \delta[n] \tag{3.14}$$

$$h_{0vh}(nT) = 0 \tag{3.15}$$

$$h_{0hv}(nT) = 0 \quad (3.16)$$

where

$$\delta[n] = \begin{Bmatrix} 1, & n = 0 \\ 0, & n \neq 0 \end{Bmatrix}. \quad (3.17)$$

Since there are L co-channels, in order to achieve zero CCI, every equalized combined co-channel impulse response, from the first to the Lth co-channel, must all be zero to achieve zero CCI. That is:

$$h_{1vv}(nT) = 0 \quad (3.18)$$

$$h_{1vh}(nT) = 0 \quad (3.19)$$

$$h_{1hv}(nT) = 0 \quad (3.20)$$

$$h_{1hh}(nT) = 0 \quad (3.21)$$

$$h_{2vv}(nT) = 0 \quad (3.22)$$

$$h_{2vh}(nT) = 0 \quad (3.23)$$

$$h_{2hv}(nT) = 0 \quad (3.24)$$

$$h_{2hh}(nT) = 0 \quad (3.25)$$

$$\vdots$$

$$h_{Lvv}(nT) = 0 \quad (3.26)$$

$$h_{Lvh}(nT) = 0 \quad (3.27)$$

$$h_{Lhv}(nT) = 0 \quad (3.28)$$

$$h_{Lhh}(nT) = 0 \quad (3.29)$$

where the subscript "1" in equations (3.18) to (3.21) denotes the first co-channel, the subscript "2" in equations (3.22) to (3.25) denotes the second co-channel, and the subscript "L" in equations (3.26) to (3.29) denotes the Lth co-channel.

The conditions in equations (3.13) and (3.14) are shown in Fig. 3.1. The conditions in equations (3.15), (3.16), and (3.18) to (3.29) are shown in Fig. 3.2 and Fig. 3.3.

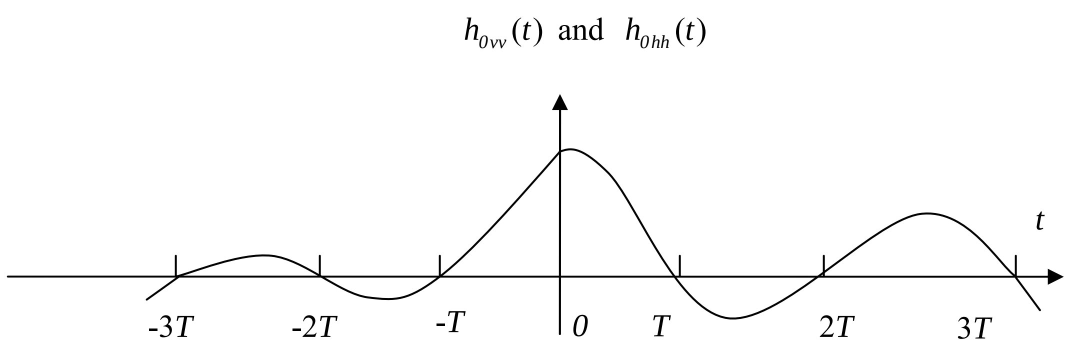


Fig. 3.1 Equalized Combined Channel:
Vertical-to-Vertical and Horizontal-to-Horizontal

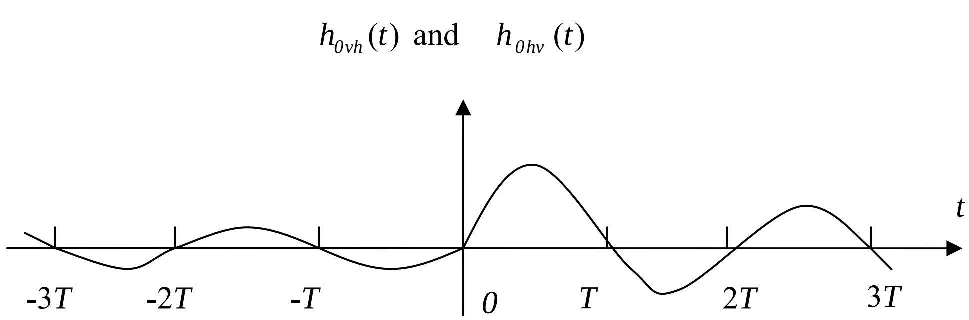


Fig. 3.2 Equalized Combined Channel:
Vertical-to-Horizontal and Horizontal-to-Vertical

$h_{Lvv}(t), h_{Lvh}(t), h_{Lhv}(t)$ and $h_{Lhh}(t)$

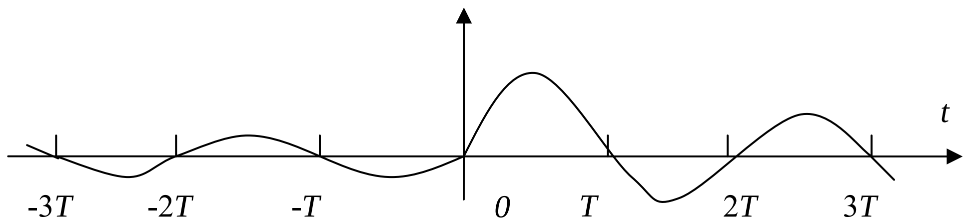


Fig. 3.3 Equalized Combined Co-channels

Equations (3.13) to (3.16) can be expressed in the frequency domain as:

$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} H_{0vv}(f+\frac{k}{T}) = 1 \quad (3.30)$$

$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} H_{0hh}(f+\frac{k}{T}) = 1 \quad (3.31)$$

$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} H_{0vh}(f+\frac{k}{T}) = 0 \quad (3.32)$$

$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} H_{0hv}(f+\frac{k}{T}) = 0 \quad (3.33)$$

Equations (3.18) to (3.29), representing the time-domain zero CCI conditions from first co-channel to L^{th} co-channel, can be expressed in frequency domain as:

$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} H_{1vv}(f+\frac{k}{T}) = 0 \quad (3.34)$$

$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} H_{1vh}(f+\frac{k}{T}) = 0 \quad (3.35)$$

$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} H_{1hv}(f+\frac{k}{T}) = 0 \quad (3.36)$$

$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} H_{1hh}(f+\frac{k}{T})=0 \qquad (3.37)$$

$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} H_{2vv}(f+\frac{k}{T})=0 \qquad (3.38)$$

$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} H_{2vh}(f+\frac{k}{T})=0 \qquad (3.39)$$

$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} H_{2hv}(f+\frac{k}{T})=0 \qquad (3.40)$$

$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} H_{2hh}(f+\frac{k}{T})=0 \qquad (3.41)$$

$$\vdots$$

$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} H_{Lvv}(f+\frac{k}{T})=0 \qquad (3.42)$$

$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} H_{Lvh}(f+\frac{k}{T})=0 \qquad (3.43)$$

$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} H_{Lhv}(f+\frac{k}{T})=0 \qquad (3.44)$$

$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} H_{Lhh}(f+\frac{k}{T})=0 \qquad (3.45)$$

where f is from a set of real numbers, k is from a set of integers.

Substituting $H_{0vv}(f)$, $H_{0vh}(f)$, $H_{0hv}(f)$ and $H_{0hh}(f)$ from equation (3.6) into equations (3.30), (3.31), (3.32) and (3.33) gives:

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{0vv}(f+\frac{k}{T})R_{vv}(f+\frac{k}{T})+\Phi_{0vh}(f+\frac{k}{T})R_{hv}(f+\frac{k}{T})\right]=T \qquad (3.46)$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{0hh}(f+\frac{k}{T})R_{hh}(f+\frac{k}{T})+\Phi_{0hv}(f+\frac{k}{T})R_{vh}(f+\frac{k}{T})\right]=T \tag{3.47}$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{0vv}(f+\frac{k}{T})R_{vh}(f+\frac{k}{T})+\Phi_{0vh}(f+\frac{k}{T})R_{hh}(f+\frac{k}{T})\right]=0 \tag{3.48}$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{0hh}(f+\frac{k}{T})R_{hv}(f+\frac{k}{T})+\Phi_{0hv}(f+\frac{k}{T})R_{vv}(f+\frac{k}{T})\right]=0 \tag{3.49}$$

Consider equation (3.9), equations from (3.34) to (3.45) representing the first co-channel to the L th co-channel can be expressed in the similar forms as:

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{1vv}(f+\frac{k}{T})R_{vv}(f+\frac{k}{T})+\Phi_{1vh}(f+\frac{k}{T})R_{hv}(f+\frac{k}{T})\right]=0 \tag{3.50}$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{1vv}(f+\frac{k}{T})R_{vh}(f+\frac{k}{T})+\Phi_{1vh}(f+\frac{k}{T})R_{hh}(f+\frac{k}{T})\right]=0 \tag{3.51}$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{1hh}(f+\frac{k}{T})R_{hv}(f+\frac{k}{T})+\Phi_{1hv}(f+\frac{k}{T})R_{vv}(f+\frac{k}{T})\right]=0 \tag{3.52}$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{1hh}(f+\frac{k}{T})R_{hh}(f+\frac{k}{T})+\Phi_{1hv}(f+\frac{k}{T})R_{vh}(f+\frac{k}{T})\right]=0 \tag{3.53}$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{2vv}(f+\frac{k}{T})R_{vv}(f+\frac{k}{T})+\Phi_{2vh}(f+\frac{k}{T})R_{hv}(f+\frac{k}{T})\right]=0 \tag{3.54}$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{2vv}(f+\frac{k}{T})R_{vh}(f+\frac{k}{T})+\Phi_{2vh}(f+\frac{k}{T})R_{hh}(f+\frac{k}{T})\right]=0 \tag{3.55}$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{2hh}(f+\frac{k}{T})R_{hv}(f+\frac{k}{T})+\Phi_{2hv}(f+\frac{k}{T})R_{vv}(f+\frac{k}{T})\right]=0 \tag{3.56}$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{2hh}(f+\frac{k}{T})R_{hh}(f+\frac{k}{T})+\Phi_{2hv}(f+\frac{k}{T})R_{vh}(f+\frac{k}{T})\right]=0 \tag{3.57}$$

$$\vdots$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{Lvv}(f+\frac{k}{T})R_{vv}(f+\frac{k}{T})+\Phi_{Lvh}(f+\frac{k}{T})R_{hv}(f+\frac{k}{T})\right]=0 \qquad (3.58)$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{Lvv}(f+\frac{k}{T})R_{vh}(f+\frac{k}{T})+\Phi_{Lvh}(f+\frac{k}{T})R_{hh}(f+\frac{k}{T})\right]=0 \qquad (3.59)$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{Lhh}(f+\frac{k}{T})R_{hv}(f+\frac{k}{T})+\Phi_{Lhv}(f+\frac{k}{T})R_{vv}(f+\frac{k}{T})\right]=0 \qquad (3.60)$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{Lhh}(f+\frac{k}{T})R_{hh}(f+\frac{k}{T})+\Phi_{Lhv}(f+\frac{k}{T})R_{vh}(f+\frac{k}{T})\right]=0 \qquad (3.61)$$

Without the knowledge of frequency responses of combined channel and combined co-channels, it is difficult to find the frequency responses of the equalizer to suppress the interference based on equations from (3.46) to (3.61). Therefore, we need to isolate the problem into different situations, or to see how the wide relative bandwidth could provide the flexibility to the linear equalizer to suppress a larger number of interferers.

We will start by analyzing the case where there is no co-channel and with zero percent excess bandwidth. Then, the bandwidth will be gradually increased when there is one or more co-channels present. The result of the ability to suppress a larger number of interferers will be generalized.

3.3.2 Dual Polarization with No Interferer

First, if the combined channel is strictly bandlimited to $1/(2T)$, this means:

$$\Phi_{0vv}(f)=0$$

$$\Phi_{0vh}(f) = 0$$

$$\Phi_{0hv}(f) = 0$$

$$\Phi_{0hh}(f) = 0, \quad \text{for } |f| \geq 1/(2T) \tag{3.62}$$

Fig. 3.4 illustrates this situation.

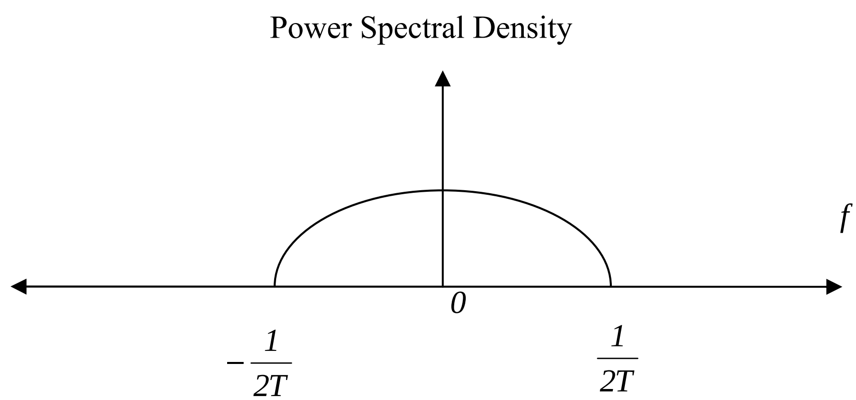


Fig. 3.4 Bandlimited from $-1/(2T)$ to $1/(2T)$

Equations (3.46), (3.47), (3.48) and (3.49) reduce to:

$$\Phi_{0vv}(f)R_{vv}(f) + \Phi_{0vh}(f)R_{hv}(f) = T \tag{3.63}$$

$$\Phi_{0hh}(f)R_{hh}(f) + \Phi_{0hv}(f)R_{vh}(f) = T \tag{3.64}$$

$$\Phi_{0vv}(f)R_{vh}(f) + \Phi_{0vh}(f)R_{hh}(f) = 0 \tag{3.65}$$

$$\Phi_{0hh}(f)R_{hv}(f) + \Phi_{0hv}(f)R_{vv}(f) = 0 \tag{3.66}$$

for $|f| \leq 1/(2T)$.

For a solution to exist in the range $|f| \leq 1/(2T)$, the number of equations must be less than or equal to the number of unknowns. Since there are four unknowns, $R_{vv}(f)$, $R_{vh}(f)$, $R_{hv}(f)$ and $R_{hh}(f)$, also four equations, (3.63), (3.64), (3.65) and (3.66), a unique solution exists. Therefore, the CPI, introduced by $\Phi_{0vh}(f)$ and $\Phi_{0hv}(f)$, and ISI can be suppressed but no co-channel interferers can be suppressed by the generalized zero-forcing equalizer.

3.3.3 Dual Polarization with One Co-channel Interferer

Consider the combined channel and combined co-channel being strictly bandlimited to $1/(2T)$, again, equations (3.46), (3.47), (3.48) and (3.49) reduce to:

$$\Phi_{0vv}(f)R_{vv}(f)+\Phi_{0vh}(f)R_{hv}(f)=T \qquad (3.63)$$

$$\Phi_{0hh}(f)R_{hh}(f)+\Phi_{0hv}(f)R_{vh}(f)=T \qquad (3.64)$$

$$\Phi_{0vv}(f)R_{vh}(f)+\Phi_{0vh}(f)R_{hh}(f)=0 \qquad (3.65)$$

$$\Phi_{0hh}(f)R_{hv}(f)+\Phi_{0hv}(f)R_{vv}(f)=0, \qquad (3.66)$$

for $|f|\le 1/(2T)$.

There is also one co-channel interferer; equations (3.50), (3.51), (3.52) and (3.53) reduce to:

$$\Phi_{1vv}(f)R_{vv}(f)+\Phi_{1vh}(f)R_{hv}(f)=0 \qquad (3.67)$$

$$\Phi_{1vv}(f)R_{vh}(f)+\Phi_{1vh}(f)R_{hh}(f)=0 \qquad (3.68)$$

$$\Phi_{1hh}(f)R_{hv}(f)+\Phi_{1hv}(f)R_{vv}(f)=0 \qquad (3.69)$$

$$\Phi_{1hh}(f)R_{hh}(f)+\Phi_{1hv}(f)R_{vh}(f)=0, \qquad (3.70)$$

for $|f|\le 1/(2T)$.

There are four unknowns, $R_{vv}(f)$, $R_{vh}(f)$, $R_{hv}(f)$ and $R_{hh}(f)$, and eight equations, (3.38) to (3.45). In general, for random channels, the number of equations is greater than the number of unknowns. Therefore, the solution does not exist in this case.

If the combined channel and combined co-channel are strictly bandlimited to $2/(2T)$, then equations (3.46), (3.47), (3.48) and (3.49) reduce to:

$$\Phi_{0vv}(f-\frac{1}{T})R_{vv}(f-\frac{1}{T})+\Phi_{0vh}(f-\frac{1}{T})R_{hv}(f-\frac{1}{T})+\Phi_{0vv}(f)R_{vv}(f)+\Phi_{0vh}(f)R_{hv}(f)=T \qquad (3.71)$$

$$\Phi_{0hv}(f-\frac{1}{T})R_{vh}(f-\frac{1}{T})+\Phi_{0hh}(f-\frac{1}{T})R_{hh}(f-\frac{1}{T})+\Phi_{0hv}(f)R_{vh}(f)+\Phi_{0hh}(f)R_{hh}(f)=T \qquad (3.72)$$

$$\Phi_{0vv}(f-\frac{1}{T})R_{vh}(f-\frac{1}{T})+\Phi_{0vh}(f-\frac{1}{T})R_{hh}(f-\frac{1}{T})+\Phi_{0vv}(f)R_{vh}(f)+\Phi_{0vh}(f)R_{hh}(f)=0 \qquad (3.73)$$

$$\Phi_{0hv}(f-\frac{1}{T})R_{vv}(f-\frac{1}{T})+\Phi_{0hh}(f-\frac{1}{T})R_{hv}(f-\frac{1}{T})+\Phi_{0hv}(f)R_{vv}(f)+\Phi_{0hh}(f)R_{hv}(f)=0 \qquad (3.74)$$

for $0<f<1/(2T)$. Fig. 3.5 illustrates this situation.

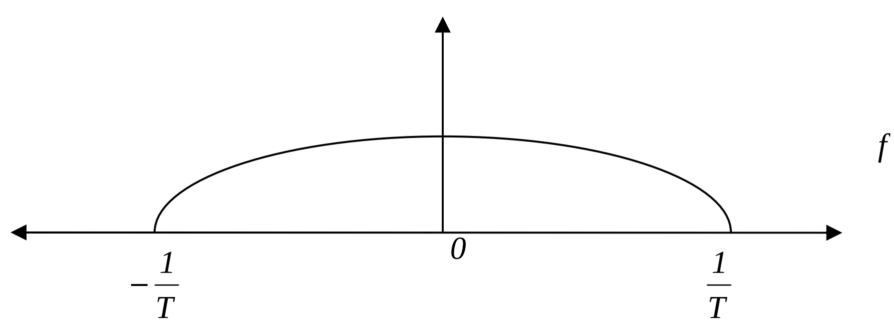


Fig. 3.5 Bandlimited from $-1/T$ to $1/T$

Since there is only one co-channel interferer, $L=1$, therefore equations (3.50), (3.51), (3.52) and (3.53) reduce to:

$$\Phi_{1vv}(f-\frac{1}{T})R_{vv}(f-\frac{1}{T})+\Phi_{1vh}(f-\frac{1}{T})R_{hv}(f-\frac{1}{T})+\Phi_{1vv}(f)R_{vv}(f)+\Phi_{1vh}(f)R_{hv}(f)=0$$

(3.75)

$$\Phi_{1vv}(f-\frac{1}{T})R_{vh}(f-\frac{1}{T})+\Phi_{1vh}(f-\frac{1}{T})R_{hh}(f-\frac{1}{T})+\Phi_{1vv}(f)R_{vh}(f)+\Phi_{1vh}(f)R_{hh}(f)=0$$

(3.76)

$$\Phi_{1hv}(f-\frac{1}{T})R_{vv}(f-\frac{1}{T})+\Phi_{1hh}(f-\frac{1}{T})R_{hv}(f-\frac{1}{T})+\Phi_{1hv}(f)R_{vv}(f)+\Phi_{1hh}(f)R_{hv}(f)=0$$

(3.77)

$$\Phi_{1hv}(f-\frac{1}{T})R_{vh}(f-\frac{1}{T})+\Phi_{1hh}(f-\frac{1}{T})R_{hh}(f-\frac{1}{T})+\Phi_{1hv}(f)R_{vh}(f)+\Phi_{1hh}(f)R_{hh}(f)=0$$

(3.78)

for $0 < f < 1/(2T)$.

Consider equations from (3.71) to (3.78), there are eight unknowns, $R_{vv}(f)$, $R_{vh}(f)$, $R_{hv}(f)$, $R_{hh}(f)$, $R_{vv}(f-\frac{1}{T})$, $R_{vh}(f-\frac{1}{T})$, $R_{hv}(f-\frac{1}{T})$, and $R_{hh}(f-\frac{1}{T})$, also eight equations. The number of unknowns equals the number of equations. Therefore, in general, one unique solution exists. One interferer, along with ISI, CPI can be completely suppressed by the generalized zero-forcing equalizer when the bandwidths are $1/T$.

3.3.4 Dual Polarization with Two Co-channel Interferers

Since there are two co-channel interferers, $L = 2$, equations (3.50) to (3.57) must be satisfied to achieve zero CCI introduced by the first and the second interferer.

If the combined channel and combined co-channel are bandlimited to $1/(2T)$, consider the first co-channel, equations (3.50) to (3.53) become:

$$\Phi_{1vv}(f)R_{vv}(f)+\Phi_{1vh}(f)R_{hv}(f)=0 \qquad (3.79)$$

$$\Phi_{1vv}(f)R_{vh}(f)+\Phi_{1vh}(f)R_{hh}(f)=0 \quad (3.80)$$

$$\Phi_{1hh}(f)R_{hv}(f)+\Phi_{1hv}(f)R_{vv}(f)=0 \quad (3.81)$$

$$\Phi_{1hh}(f)R_{hh}(f)+\Phi_{1hv}(f)R_{vh}(f)=0 \quad (3.82)$$

The equations, (3.54) to (3.57), of the second co-channel, reduce to:

$$\Phi_{2vv}(f)R_{vv}(f)+\Phi_{2vh}(f)R_{hv}(f)=0 \quad (3.83)$$

$$\Phi_{2vv}(f)R_{vh}(f)+\Phi_{2vh}(f)R_{hh}(f)=0 \quad (3.84)$$

$$\Phi_{2hh}(f)R_{hv}(f)+\Phi_{2hv}(f)R_{vv}(f)=0 \quad (3.85)$$

$$\Phi_{2hh}(f)R_{hh}(f)+\Phi_{2hv}(f)R_{vh}(f)=0 \quad (3.86)$$

for $|f|\le 1/(2T)$.

There are also four equations, (3.63) to (3.66), for the data transmitted.

$$\Phi_{0vv}(f)R_{vv}(f)+\Phi_{0vh}(f)R_{hv}(f)=T \quad (3.63)$$

$$\Phi_{0hh}(f)R_{hh}(f)+\Phi_{0hv}(f)R_{vh}(f)=T \quad (3.64)$$

$$\Phi_{0vv}(f)R_{vh}(f)+\Phi_{0vh}(f)R_{hh}(f)=0 \quad (3.65)$$

$$\Phi_{0hh}(f)R_{hv}(f)+\Phi_{0hv}(f)R_{vv}(f)=0 \quad (3.66)$$

for $|f|\le 1/(2T)$.

Consider equations (3.63) to (3.66) and (3.79) to (3.86), there are total of twelve equations. There are four unknowns, $R_{vv}(f)$, $R_{vh}(f)$, $R_{hv}(f)$ and $R_{hh}(f)$. The number of unknowns is less than the number of equations. Therefore, the solution does not exist. The linear equalizer cannot suppress CCI and CPI introduced by co-channel interferers.

If the combined channel and combined co-channel are bandlimited to $2/(2T)$, equations (3.50) to (3.53) reduce to the following equations for the first co-channel:

$$\Phi_{1vv}(f-\frac{1}{T})R_{vv}(f-\frac{1}{T})+\Phi_{1vh}(f-\frac{1}{T})R_{hv}(f-\frac{1}{T})+\Phi_{1vv}(f)R_{vv}(f)+\Phi_{1vh}(f)R_{hv}(f)=0$$

(3.87)

$$\Phi_{1vv}(f-\frac{1}{T})R_{vh}(f-\frac{1}{T})+\Phi_{1vh}(f-\frac{1}{T})R_{hh}(f-\frac{1}{T})+\Phi_{1vv}(f)R_{vh}(f)+\Phi_{1vh}(f)R_{hh}(f)=0$$

(3.88)

$$\Phi_{1hv}(f-\frac{1}{T})R_{vv}(f-\frac{1}{T})+\Phi_{1hh}(f-\frac{1}{T})R_{hv}(f-\frac{1}{T})+\Phi_{1hv}(f)R_{vv}(f)+\Phi_{1hh}(f)R_{hv}(f)=0$$

(3.89)

$$\Phi_{1hv}(f-\frac{1}{T})R_{vh}(f-\frac{1}{T})+\Phi_{1hh}(f-\frac{1}{T})R_{hh}(f-\frac{1}{T})+\Phi_{1hv}(f)R_{vh}(f)+\Phi_{1hh}(f)R_{hh}(f)=0$$

(3.90)

for $0<f<1/(2T)$.

Equations (3.54) to (3.57) reduce to the following for the second co-channel:

$$\Phi_{2vv}(f-\frac{1}{T})R_{vv}(f-\frac{1}{T})+\Phi_{2vh}(f-\frac{1}{T})R_{hv}(f-\frac{1}{T})+\Phi_{2vv}(f)R_{vv}(f)+\Phi_{2vh}(f)R_{hv}(f)=0$$

(3.91)

$$\Phi_{2vv}(f-\frac{1}{T})R_{vh}(f-\frac{1}{T})+\Phi_{2vh}(f-\frac{1}{T})R_{hh}(f-\frac{1}{T})+\Phi_{2vv}(f)R_{vh}(f)+\Phi_{2vh}(f)R_{hh}(f)=0$$

(3.92)

$$\Phi_{2hv}(f-\frac{1}{T})R_{vv}(f-\frac{1}{T})+\Phi_{2hh}(f-\frac{1}{T})R_{hv}(f-\frac{1}{T})+\Phi_{2hv}(f)R_{vv}(f)+\Phi_{2hh}(f)R_{hv}(f)=0$$

(3.93)

$$\Phi_{2hv}(f-\frac{1}{T})R_{vh}(f-\frac{1}{T})+\Phi_{2hh}(f-\frac{1}{T})R_{hh}(f-\frac{1}{T})+\Phi_{2hv}(f)R_{vh}(f)+\Phi_{2hh}(f)R_{hh}(f)=0$$

(3.94)

for $0 < f < 1/(2T)$.

Also we need to include four equations for the channel that transmits the data:

$$\Phi_{0vv}(f-\frac{1}{T})R_{vv}(f-\frac{1}{T})+\Phi_{0vh}(f-\frac{1}{T})R_{hv}(f-\frac{1}{T})+\Phi_{0vv}(f)R_{vv}(f)+\Phi_{0vh}(f)R_{hv}(f)=T$$

(3.71)

$$\Phi_{0hv}(f-\frac{1}{T})R_{vh}(f-\frac{1}{T})+\Phi_{0hh}(f-\frac{1}{T})R_{hh}(f-\frac{1}{T})+\Phi_{0hv}(f)R_{vh}(f)+\Phi_{0hh}(f)R_{hh}(f)=T$$

(3.72)

$$\Phi_{0vv}(f-\frac{1}{T})R_{vh}(f-\frac{1}{T})+\Phi_{0vh}(f-\frac{1}{T})R_{hh}(f-\frac{1}{T})+\Phi_{0vv}(f)R_{vh}(f)+\Phi_{0vh}(f)R_{hh}(f)=0$$

(3.73)

$$\Phi_{0hv}(f-\frac{1}{T})R_{vv}(f-\frac{1}{T})+\Phi_{0hh}(f-\frac{1}{T})R_{hv}(f-\frac{1}{T})+\Phi_{0hv}(f)R_{vv}(f)+\Phi_{0hh}(f)R_{hv}(f)=0$$

(3.74)

for $0 < f < 1/(2T)$.

Consider equations (3.71) to (3.74) and equations (3.87) to (3.94), there are twelve equations and eight unknowns, $R_{vv}(f)$, $R_{vh}(f)$, $R_{hv}(f)$, $R_{hh}(f)$, $R_{vv}(f-\frac{1}{T})$, $R_{vh}(f-\frac{1}{T})$, $R_{hv}(f-\frac{1}{T})$, and $R_{hh}(f-\frac{1}{T})$. The number of unknowns is less than the number of equations. In general, the solution does not exist. The equalizer cannot suppress the co-channels introduced interference.

So far, it is obvious that an increase in bandwidth will increase the degree of freedom, and the number of unknowns. If the combined channel and combined co-channels are bandlimited to $3/(2T)$, consider the following Fig. 3.6.

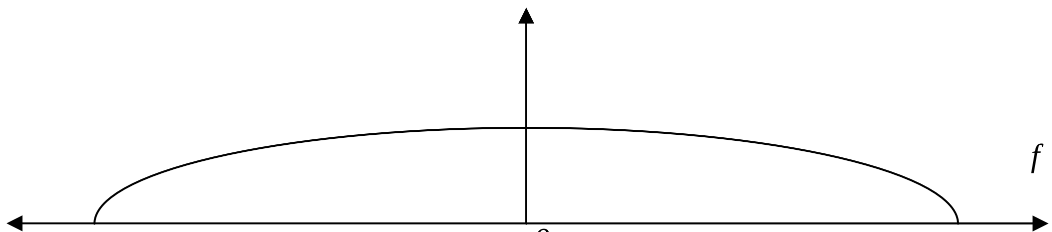


Fig. 3.6 Bandlimited from $-3/2T$ **to** $3/2T$

Equations (3.46) to (3.49) reduce to:

$$\Phi_{0vv}(f-\frac{1}{T})R_{vv}(f-\frac{1}{T})+\Phi_{0vh}(f-\frac{1}{T})R_{hv}(f-\frac{1}{T})+\Phi_{0vv}(f)R_{vv}(f)+\Phi_{0vh}(f)R_{hv}(f)+$$

$$+\Phi_{0vv}(f+\frac{1}{T})R_{vv}(f+\frac{1}{T})+\Phi_{0vh}(f+\frac{1}{T})R_{hv}(f+\frac{1}{T})=T \qquad (3.95)$$

$$\Phi_{0hv}(f-\frac{1}{T})R_{vh}(f-\frac{1}{T})+\Phi_{0hh}(f-\frac{1}{T})R_{hh}(f-\frac{1}{T})+\Phi_{0hv}(f)R_{vh}(f)+\Phi_{0hh}(f)R_{hh}(f)+$$

$$+\Phi_{0hv}(f+\frac{1}{T})R_{vh}(f+\frac{1}{T})+\Phi_{0hh}(f+\frac{1}{T})R_{hh}(f+\frac{1}{T})=T \qquad (3.96)$$

$$\Phi_{0vv}(f-\frac{1}{T})R_{vh}(f-\frac{1}{T})+\Phi_{0vh}(f-\frac{1}{T})R_{hh}(f-\frac{1}{T})+\Phi_{0vv}(f)R_{vh}(f)+\Phi_{0vh}(f)R_{hh}(f)+$$

$$+\Phi_{0vv}(f+\frac{1}{T})R_{vh}(f+\frac{1}{T})+\Phi_{0vh}(f+\frac{1}{T})R_{hh}(f+\frac{1}{T})=0 \qquad (3.97)$$

$$\Phi_{0hv}(f-\frac{1}{T})R_{vv}(f-\frac{1}{T})+\Phi_{0hh}(f-\frac{1}{T})R_{hv}(f-\frac{1}{T})+\Phi_{0hv}(f)R_{vv}(f)+\Phi_{0hh}(f)R_{hv}(f)+$$

$$+\Phi_{0hv}(f+\frac{1}{T})R_{vv}(f+\frac{1}{T})+\Phi_{0hh}(f+\frac{1}{T})R_{hv}(f+\frac{1}{T})=0, \qquad (3.98)$$

for $0 < f < 1/(2T)$.

Equations (3.50) to (3.53) reduce to the following four equations for the first co-channel:

$$\Phi_{1vv}(f-\frac{1}{T})R_{vv}(f-\frac{1}{T})+\Phi_{1vh}(f-\frac{1}{T})R_{hv}(f-\frac{1}{T})+\Phi_{1vv}(f)R_{vv}(f)+\Phi_{1vh}(f)R_{hv}(f)+$$

$$+\Phi_{1vv}(f+\frac{1}{T})R_{vv}(f+\frac{1}{T})+\Phi_{1vh}(f+\frac{1}{T})R_{hv}(f+\frac{1}{T})=0 \qquad (3.99)$$

$$\Phi_{1vv}(f-\frac{1}{T})R_{vh}(f-\frac{1}{T})+\Phi_{1vh}(f-\frac{1}{T})R_{hh}(f-\frac{1}{T})+\Phi_{1vv}(f)R_{vh}(f)+\Phi_{1vh}(f)R_{hh}(f)+$$

$$+\Phi_{1vv}(f+\frac{1}{T})R_{vh}(f+\frac{1}{T})+\Phi_{1vh}(f+\frac{1}{T})R_{hh}(f+\frac{1}{T})=0 \qquad (3.100)$$

$$\Phi_{1hv}(f-\frac{1}{T})R_{vv}(f-\frac{1}{T})+\Phi_{1hh}(f-\frac{1}{T})R_{hv}(f-\frac{1}{T})+\Phi_{1hv}(f)R_{vv}(f)+\Phi_{1hh}(f)R_{hv}(f)+$$

$$+\Phi_{1hv}(f+\frac{1}{T})R_{vv}(f+\frac{1}{T})+\Phi_{1hh}(f+\frac{1}{T})R_{hv}(f+\frac{1}{T})=0 \qquad (3.101)$$

$$\Phi_{1hv}(f-\frac{1}{T})R_{vh}(f-\frac{1}{T})+\Phi_{1hh}(f-\frac{1}{T})R_{hh}(f-\frac{1}{T})+\Phi_{1hv}(f)R_{vh}(f)+\Phi_{1hh}(f)R_{hh}(f)+$$

$$+\Phi_{1hv}(f+\frac{1}{T})R_{vh}(f+\frac{1}{T})+\Phi_{1hh}(f+\frac{1}{T})R_{hh}(f+\frac{1}{T})=0 \qquad (3.102)$$

Similarly, equations (3.54) to (3.57) reduce to the following equations for the second co-channel:

$$\Phi_{2vv}(f-\frac{1}{T})R_{vv}(f-\frac{1}{T})+\Phi_{2vh}(f-\frac{1}{T})R_{hv}(f-\frac{1}{T})+\Phi_{2vv}(f)R_{vv}(f)+\Phi_{2vh}(f)R_{hv}(f)+$$

$$+\Phi_{2vv}(f+\frac{1}{T})R_{vv}(f+\frac{1}{T})+\Phi_{2vh}(f+\frac{1}{T})R_{hv}(f+\frac{1}{T})=0 \qquad (3.103)$$

$$\Phi_{2vv}(f-\frac{1}{T})R_{vh}(f-\frac{1}{T})+\Phi_{2vh}(f-\frac{1}{T})R_{hh}(f-\frac{1}{T})+\Phi_{2vv}(f)R_{vh}(f)+\Phi_{2vh}(f)R_{hh}(f)+$$

$$+\Phi_{2vv}(f+\frac{1}{T})R_{vh}(f+\frac{1}{T})+\Phi_{2vh}(f+\frac{1}{T})R_{hh}(f+\frac{1}{T})=0 \qquad (3.104)$$

$$\Phi_{2hv}(f-\frac{1}{T})R_{vv}(f-\frac{1}{T})+\Phi_{2hh}(f-\frac{1}{T})R_{hv}(f-\frac{1}{T})+\Phi_{2hv}(f)R_{vv}(f)+\Phi_{2hh}(f)R_{hv}(f)+$$

$$+\Phi_{2hv}(f+\frac{1}{T})R_{vv}(f+\frac{1}{T})+\Phi_{2hh}(f+\frac{1}{T})R_{hv}(f+\frac{1}{T})=0 \qquad (3.105)$$

$$\Phi_{2hv}(f-\frac{1}{T})R_{vh}(f-\frac{1}{T})+\Phi_{2hh}(f-\frac{1}{T})R_{hh}(f-\frac{1}{T})+\Phi_{2hv}(f)R_{vh}(f)+\Phi_{2hh}(f)R_{hh}(f)+$$

$$+\Phi_{2hv}(f+\frac{1}{T})R_{vh}(f+\frac{1}{T})+\Phi_{2hh}(f+\frac{1}{T})R_{hh}(f+\frac{1}{T})=0, \qquad (3.106)$$

for $0 < f < 1/(2T)$.

There are twelve equations, from (3.95) to (3.106) and there are twelve unknowns, $R_{vv}(f)$, $R_{vh}(f)$, $R_{hv}(f)$, $R_{hh}(f)$, $R_{vv}(f-\frac{1}{T})$, $R_{vh}(f-\frac{1}{T})$, $R_{hv}(f-\frac{1}{T})$, $R_{hh}(f-\frac{1}{T})$, $R_{vv}(f+\frac{1}{T})$, $R_{vh}(f+\frac{1}{T})$, $R_{hv}(f+\frac{1}{T})$ and $R_{hh}(f+\frac{1}{T})$. The number of unknowns is equal to the number of equations, therefore, the solution exists. The generalized zero-forcing equalizer can suppress ISI, CPI and two CCI introduced by the other two co-channels under this condition when the bandwidth is $3/(2T)$.

3.3.5 Dual Polarization with Multiple Co-channel Interferers

If there are L co-channels, according to the analyses in the previous cases, for a solution to exist, the combined channel and the combined co-channels have to be bandlimited to not less than $(L+1)/2T$. The number of equations and the number of unknowns are twelve each under this condition. A unique solution exists for a generalized zero-forcing equalizer to suppress all ISI, CPI and CPI.

Any increase in total bandwidth will not increase the number of equations, but will increase the number of unknowns. A solution will always exist if that is the case.

3.4 Generalized Results

The generalized zero-forcing equalizer will exist and have the ability to suppress all ISI, CPI and CCI under certain conditions. If the number of co-channels is L, the combined channel and the combined co-channels are bandlimited to $M/(2T)$, for a solution to exist, the following condition must be satisfied:

$$L \leq M \ . \tag{3.107}$$

where M is from a set of real numbers. This is the fundamental result.

In summary, every increase in total bandwidth of size equal to the symbol rate may provide the flexibility to completely suppress an additional interferer by means of a linear equalizer.

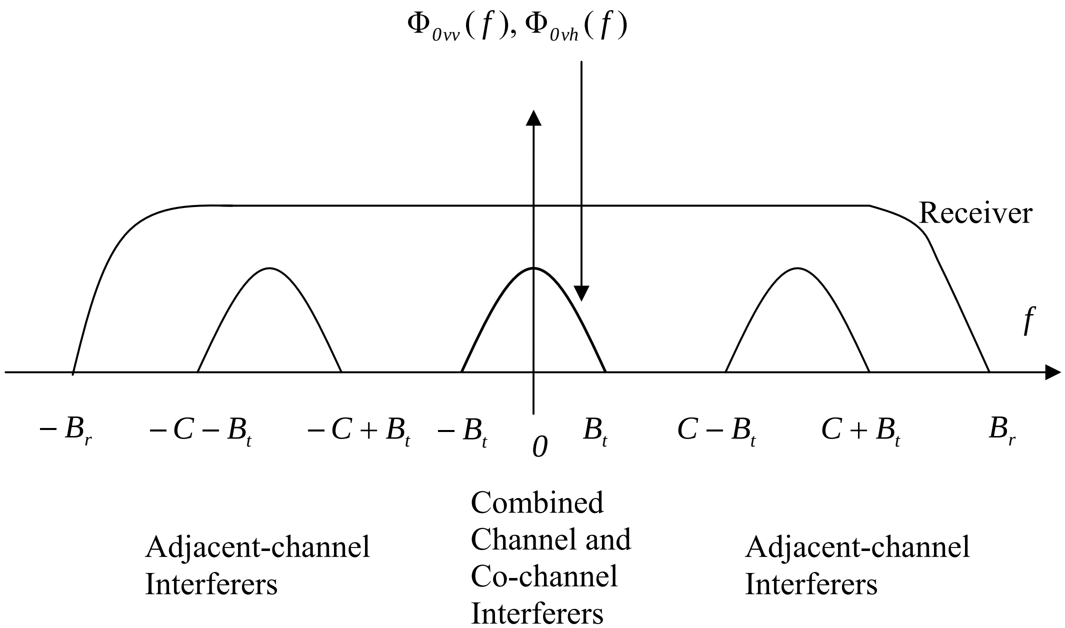
Chapter 4

Equalizer Analyses for ACI

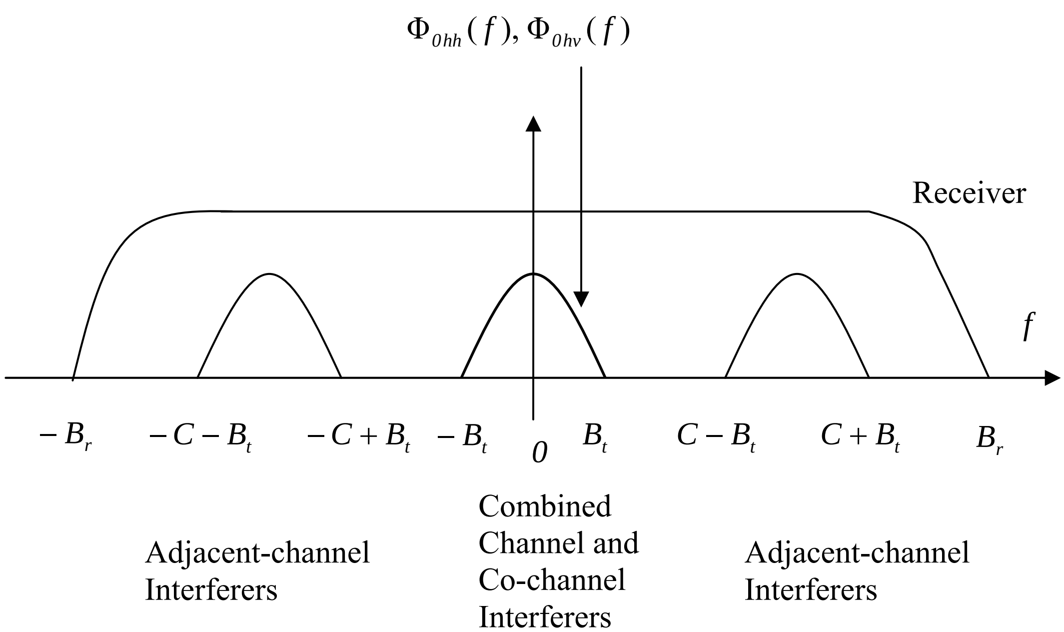
4.1 Adjacent-channel Interference

The existence of a generalized zero-forcing equalizer has been analyzed to this point, as well as the number of co-channels and transmitter bandwidth relative to symbol rate. The analyses in this chapter will include ACI, receiver bandwidth and carrier spacing.

Fig. 2.4 shows a linear equalizer which operates in the presence of CCI, CPI and L ACI interferers. The power spectral densities of the signal of the combined channel for the data transmitted, L interferers and CCI signals at the receiver filter are shown in Fig. 4.1. The combined channel for the data transmitted and the CCI signals are centered at 0 Hz. The ACI signals represented by L interferers are centered at non-zero frequencies. B_t is the transmitter bandwidth, B_r is the receiver bandwidth and C is the carrier spacing. Fig. 4.1 (a) shows the vertical-to-vertical channel and vertical-to-horizontal-channel, (b) shows the horizontal-to-horizontal channel and horizontal-to-vertical channel. If this technique is applied to optical WDMA, zero CCI is assumed.



(a) Vertical-to-Vertical and Vertical-to-Horizontal Channel



(b) Horizontal-to-Horizontal and Horizontal-to-Vertical Channel

Fig. 4.1 Power Spectral Density of the Signals After Receiver Filtering

4.2 Transmitter Bandwidth, Receiver Bandwidth and Carrier Spacing

The transmitter bandwidth, receiver bandwidth and carrier spacing are measured relative to the symbol rate, $1/T$. The normalization of these parameters are illustrated in the following subsections.

4.2.1 Transmitter Bandwidth

The normalized transmitter bandwidth relative to symbol rate, $1/T$, is shown in Fig. 4.2.

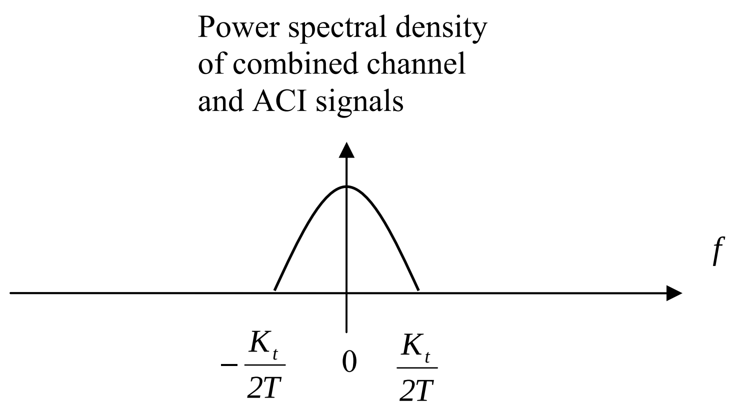


Fig. 4.2 Normalized Transmitter Bandwidth

Define the real transmitter bandwidth as W_t:

$$W_t = \frac{K_t}{2T}, \tag{4.1}$$

where K_t is from a set of real numbers, then the normalized transmitter bandwidth B_t is defined as:

$$B_t = \frac{K_t}{2}. \tag{4.2}$$

Since K_t is from a set of real numbers, therefore, B_t is also a real number. To obtain the real transmitter bandwidth, just multiply the normalized bandwidth, B_t, by the symbol rate, $1/T$.

4.2.2 Receiver Bandwidth

The definition of normalized receiver bandwidth, relative to symbol rate, $1/T$, is shown in Fig. 4.3.

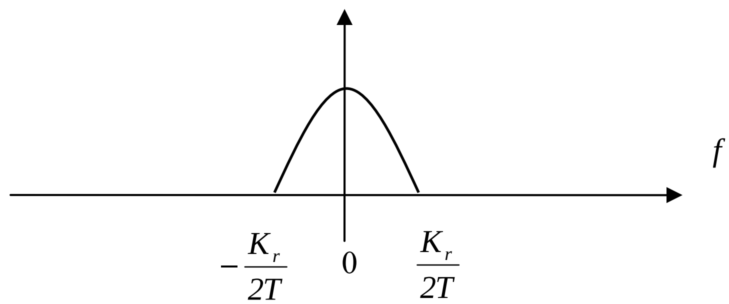


Fig. 4.3 Normalized Receiver Bandwidth

Define the real receiver bandwidth as W_r:

$$W_r = \frac{K_r}{2T}, \tag{4.3}$$

where K_r is from a set of real numbers, then the normalized receiver bandwidth B_r is defined as:

$$B_r = \frac{K_r}{2}. \tag{4.4}$$

4.2.3 Carrier Spacing

Carrier spacing relative to symbol rate is defined in a similar way.

Define the real carrier spacing as W_c:

$$W_c = \frac{K_c}{2T}, \tag{4.5}$$

where K_c is from a set of real numbers, then the normalized carrier spacing, C, relative to symbol rate, $1/T$, can be defined as:

$$C = \frac{K_c}{2}. \tag{4.6}$$

4.3 Equalizer Analysis

4.3.1 Zero ACI Conditions

Consider Fig. 2.4, in order to achieve zero ISI, CCI and ACI, the following conditions must be true:

$$h_{0vv}(nT) = \delta[n] \tag{4.7}$$

$$h_{0hh}(nT) = \delta[n] \tag{4.8}$$

$$h_{0vh}(nT) = 0 \tag{4.9}$$

$$h_{0hh}(nT) = 0 \tag{4.10}$$

Also, since there are L ACI interferers present, every interferer must satisfy zero ACI and CPI conditions, from the first channel to the Lth channel:

$$h_{1vv}(nT) = 0 \tag{4.11}$$

$$h_{1vh}(nT) = 0 \tag{4.12}$$

$$h_{1hv}(nT) = 0 \tag{4.13}$$

$$h_{1hh}(nT) = 0 \tag{4.14}$$

$$h_{2vv}(nT) = 0 \tag{4.15}$$

$$h_{2vh}(nT) = 0 \tag{4.16}$$

$$h_{2hv}(nT) = 0 \tag{4.17}$$

$$h_{2hh}(nT) = 0 \tag{4.18}$$

$$\vdots$$

$$h_{Lvv}(nT) = 0 \tag{4.19}$$

$$h_{Lvh}(nT) = 0 \tag{4.20}$$

$$h_{Lhv}(nT) = 0 \tag{4.21}$$

$$h_{Lhh}(nT) = 0 \tag{4.22}$$

where L denotes L th ACI interferers. Therefore, the following frequency domain conditions must be true:

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{0vv}(f+\frac{k}{T})R_{vv}(f+\frac{k}{T})+\Phi_{0vh}(f+\frac{k}{T})R_{hv}(f+\frac{k}{T})\right]=T \tag{4.23}$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{0hh}(f+\frac{k}{T})R_{hh}(f+\frac{k}{T})+\Phi_{0hv}(f+\frac{k}{T})R_{vh}(f+\frac{k}{T})\right]=T \tag{4.24}$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{0vv}(f+\frac{k}{T})R_{vh}(f+\frac{k}{T})+\Phi_{0vh}(f+\frac{k}{T})R_{hh}(f+\frac{k}{T})\right]=0 \tag{4.25}$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{0hh}(f+\frac{k}{T})R_{hv}(f+\frac{k}{T})+\Phi_{0hv}(f+\frac{k}{T})R_{vv}(f+\frac{k}{T})\right]=0 \tag{4.26}$$

The frequency domain expressions for the ACI channels, from the first channel to the L th channel are:

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{1vv}(f+\frac{k}{T})R_{vv}(f+\frac{k}{T})+\Phi_{1vh}(f+\frac{k}{T})R_{hv}(f+\frac{k}{T})\right]=0 \tag{4.27}$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{1vv}(f+\frac{k}{T})R_{vh}(f+\frac{k}{T})+\Phi_{1vh}(f+\frac{k}{T})R_{hh}(f+\frac{k}{T})\right]=0 \tag{4.28}$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{1hh}(f+\frac{k}{T})R_{hv}(f+\frac{k}{T})+\Phi_{1hv}(f+\frac{k}{T})R_{vv}(f+\frac{k}{T})\right]=0 \tag{4.29}$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{1hh}(f+\frac{k}{T})R_{hh}(f+\frac{k}{T})+\Phi_{1hv}(f+\frac{k}{T})R_{vh}(f+\frac{k}{T})\right]=0 \qquad (4.30)$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{2vv}(f+\frac{k}{T})R_{vv}(f+\frac{k}{T})+\Phi_{2vh}(f+\frac{k}{T})R_{hv}(f+\frac{k}{T})\right]=0 \qquad (4.31)$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{2vv}(f+\frac{k}{T})R_{vh}(f+\frac{k}{T})+\Phi_{2vh}(f+\frac{k}{T})R_{hh}(f+\frac{k}{T})\right]=0 \qquad (4.32)$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{2hh}(f+\frac{k}{T})R_{hv}(f+\frac{k}{T})+\Phi_{2hv}(f+\frac{k}{T})R_{vv}(f+\frac{k}{T})\right]=0 \qquad (4.33)$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{2hh}(f+\frac{k}{T})R_{hh}(f+\frac{k}{T})+\Phi_{2hv}(f+\frac{k}{T})R_{vh}(f+\frac{k}{T})\right]=0 \qquad (4.34)$$

$$\vdots$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{Lvv}(f+\frac{k}{T})R_{vv}(f+\frac{k}{T})+\Phi_{Lvh}(f+\frac{k}{T})R_{hv}(f+\frac{k}{T})\right]=0 \qquad (4.35)$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{Lvv}(f+\frac{k}{T})R_{vh}(f+\frac{k}{T})+\Phi_{Lvh}(f+\frac{k}{T})R_{hh}(f+\frac{k}{T})\right]=0 \qquad (4.36)$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{Lhh}(f+\frac{k}{T})R_{hv}(f+\frac{k}{T})+\Phi_{Lhv}(f+\frac{k}{T})R_{vv}(f+\frac{k}{T})\right]=0 \qquad (4.37)$$

$$\sum_{k=-\infty}^{+\infty}\left[\Phi_{Lhh}(f+\frac{k}{T})R_{hh}(f+\frac{k}{T})+\Phi_{Lhv}(f+\frac{k}{T})R_{vh}(f+\frac{k}{T})\right]=0 \qquad (4.38)$$

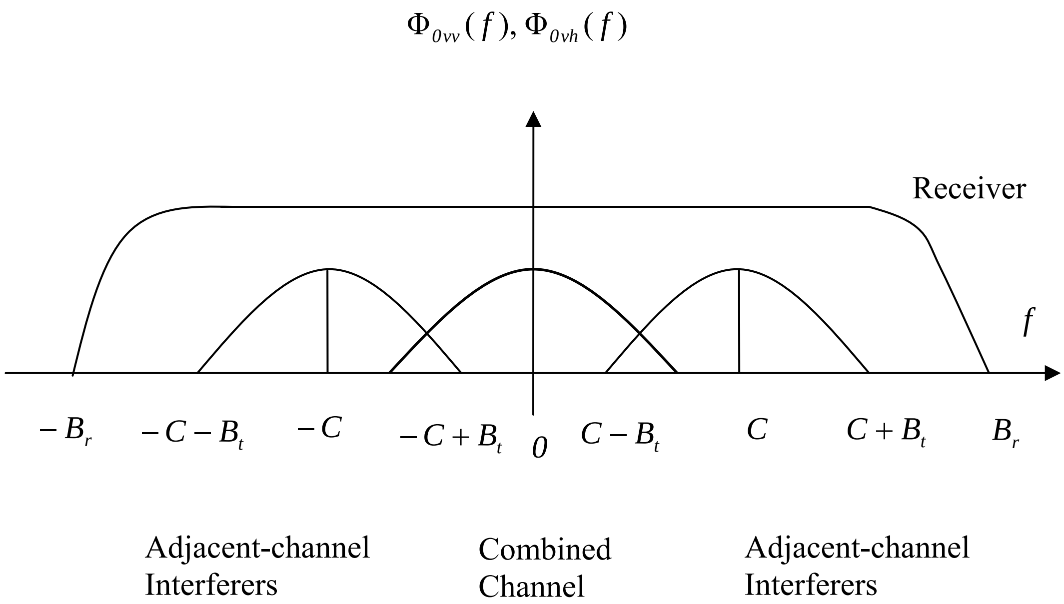
Since the ACI signals represented by L interferers are centered at non-zero frequencies, in order to analyze the relation between the number of unknowns and equations, we need to study whether ACI signals overlap with the signal of the data transmitted. Overlap occurs when $C < 2B_t$. The non-overlap case occurs when $C \geq 2B_t$ and it is shown in Fig. 4.1.

4.3.2 Non-overlap Case

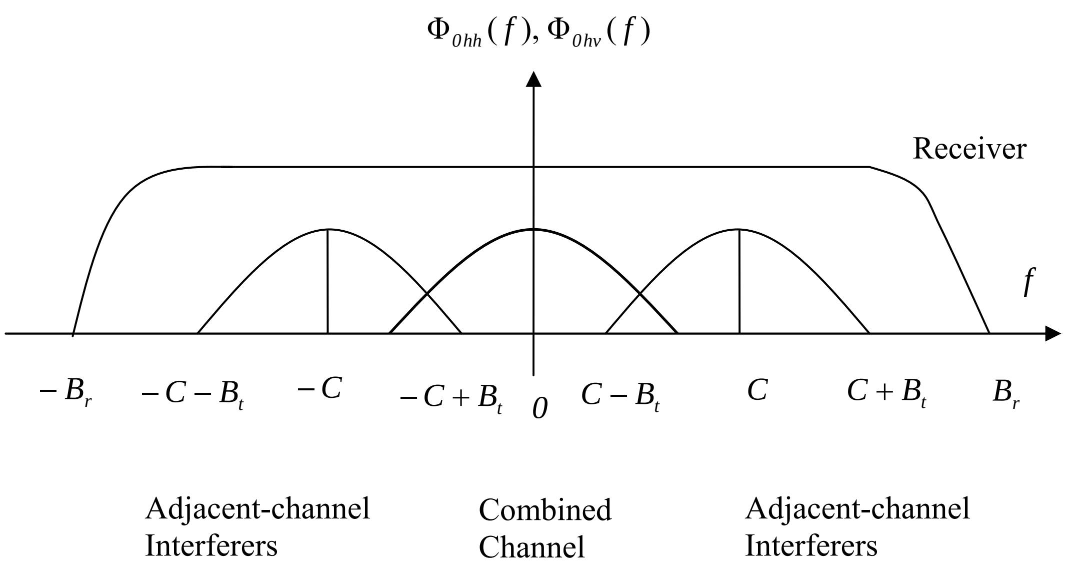
If there is no overlap, the number of adjacent interferers is zero. The equations that need to be considered are (4.23), (4.24), (4.25) and (4.26). If the bandwidth, B_t, is less than $1/(2T)$, it is impossible to completely suppress ISI and CPI of the data transmitted. If B_t is equal or greater than $1/(2T)$, from the analyses in section 3.3.2, the ISI and CPI can be completely suppressed since the number of unknowns is always equal to or greater than the number of equations. A generalized zero-forcing equalizer exists and has the ability to suppress ISI and CPI.

4.3.3 Overlap Case

Consider Fig. 4.4 where the power spectral densities of combined channel, ACI interferers presented at the receiver are illustrated. Fig. 4.4 (a) shows the vertical-to-vertical channel and vertical-to-horizontal-channel, (b) shows the horizontal-to-horizontal channel and horizontal-to-vertical channel. The combined channel for the data transmitted are centered at 0 Hz, ACI interferers overlapping with data transmitted are located at non-zero frequency C. All signals are transmitting at the same bandwidth, B_t. The same approach applied in Chapter 3 which leads to the co-channel interference solution is repeated here. The fundamental idea is to determine the degree of overlap introduced by adjacent interferers, apply zero ISI, CPI and ACI conditions and obtain the number of unknowns and the number of equations to determine whether a generalized zero-forcing equalizer exists. The analyses begin with a few random assigned values to the transmitter bandwidth, B_t, and carrier spacing, C. Also, receiver bandwidth, B_r, is assumed equal to the transmitter bandwidth, B_t, at this point.



(a) Vertical-to-Vertical and Vertical-to-Horizontal Channel



(b) Horizontal-to-Horizontal and Horizontal-to-Vertical Channel

Fig. 4.4 Power Spectral Density of the Signals after Receiver Filtering

Consider the case where $C = 2$, $B_t = 1.6$, $B_r = B_t$. This is the overlap case since $2B_t > C$ and there are two ACI channel interferers. It is illustrated in Fig. 4.5. For simplicity, only the vertical-to-vertical and vertical-to-horizontal channels are shown. In order to suppress ISI, CPI and ACI, for the channel that transmits the data, equations (4.23) to (4.26) reduce to the following four equations:

$$\Phi_{0vv}(f-\frac{1}{T})R_{vv}(f-\frac{1}{T})+\Phi_{0vh}(f-\frac{1}{T})R_{hv}(f-\frac{1}{T})+\Phi_{0vv}(f)R_{vv}(f)+\Phi_{0vh}(f)R_{hv}(f)+$$

$$+\Phi_{0vv}(f+\frac{1}{T})R_{vv}(f+\frac{1}{T})+\Phi_{0vh}(f+\frac{1}{T})R_{hv}(f+\frac{1}{T})=T \quad (4.39)$$

$$\Phi_{0hv}(f-\frac{1}{T})R_{vh}(f-\frac{1}{T})+\Phi_{0hh}(f-\frac{1}{T})R_{hh}(f-\frac{1}{T})+\Phi_{0hv}(f)R_{vh}(f)+\Phi_{0hh}(f)R_{hh}(f)+$$

$$+\Phi_{0hv}(f+\frac{1}{T})R_{vh}(f+\frac{1}{T})+\Phi_{0hh}(f+\frac{1}{T})R_{hh}(f+\frac{1}{T})=T \quad (4.40)$$

$$\Phi_{0vv}(f-\frac{1}{T})R_{vh}(f-\frac{1}{T})+\Phi_{0vh}(f-\frac{1}{T})R_{hh}(f-\frac{1}{T})+\Phi_{0vv}(f)R_{vh}(f)+\Phi_{0vh}(f)R_{hh}(f)+$$

$$+\Phi_{0vv}(f+\frac{1}{T})R_{vh}(f+\frac{1}{T})+\Phi_{0vh}(f+\frac{1}{T})R_{hh}(f+\frac{1}{T})=0 \quad (4.41)$$

$$\Phi_{0hv}(f-\frac{1}{T})R_{vv}(f-\frac{1}{T})+\Phi_{0hh}(f-\frac{1}{T})R_{hv}(f-\frac{1}{T})+\Phi_{0hv}(f)R_{vv}(f)+\Phi_{0hh}(f)R_{hv}(f)+$$

$$+\Phi_{0hv}(f+\frac{1}{T})R_{vv}(f+\frac{1}{T})+\Phi_{0hh}(f+\frac{1}{T})R_{hv}(f+\frac{1}{T})=0, \quad (4.42)$$

for $0 < f < 1/(2T)$.

The unknowns are $R_{vv}(f)$, $R_{vh}(f)$, $R_{hv}(f)$, $R_{hh}(f)$, $R_{vv}(f-\frac{1}{T})$, $R_{vh}(f-\frac{1}{T})$, $R_{hv}(f-\frac{1}{T})$, $R_{hh}(f-\frac{1}{T})$, $R_{vv}(f+\frac{1}{T})$, $R_{vh}(f+\frac{1}{T})$, $R_{hv}(f+\frac{1}{T})$ and $R_{hh}(f+\frac{1}{T})$.

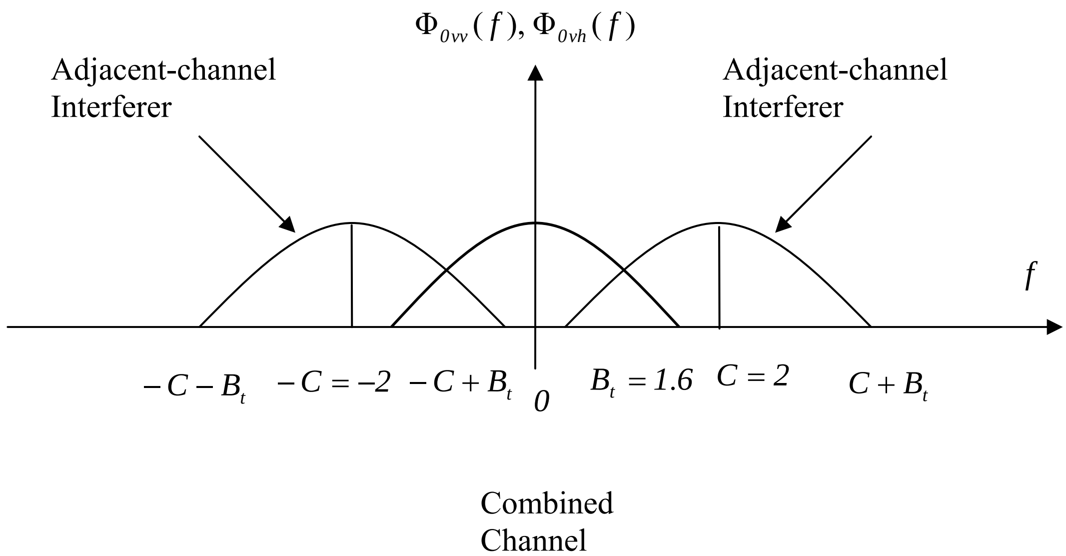


Fig. 4.5 Vertical-to-Vertical and Vertical-to-Horizontal Channel

For the first interfering ACI channel, equations (4.27) to (4.30) produce the following results:

$$\Phi_{1vv}(f-\frac{1}{T})R_{vv}(f-\frac{1}{T})+\Phi_{1vh}(f-\frac{1}{T})R_{hv}(f-\frac{1}{T})=0 \qquad (4.43)$$

$$\Phi_{1vv}(f-\frac{1}{T})R_{vh}(f-\frac{1}{T})+\Phi_{1vh}(f-\frac{1}{T})R_{hh}(f-\frac{1}{T})=0 \qquad (4.44)$$

$$\Phi_{1hh}(f-\frac{1}{T})R_{hv}(f-\frac{1}{T})+\Phi_{1hv}(f-\frac{1}{T})R_{vv}(f-\frac{1}{T})=0 \qquad (4.45)$$

$$\Phi_{1hh}(f-\frac{1}{T})R_{hh}(f-\frac{1}{T})+\Phi_{1hv}(f-\frac{1}{T})R_{vh}(f-\frac{1}{T})=0 \qquad (4.46)$$

for $0<f<1/(2T)$. Note that (f) and $(f+1/T)$ terms disappeared because they are out of the receiver bandwidth, B_r.

Similarly, equations (4.31) to (4.34) produce the following four equations:

$$\Phi_{2vv}(f+\frac{1}{T})R_{vv}(f+\frac{1}{T})+\Phi_{2vh}(f+\frac{1}{T})R_{hv}(f+\frac{1}{T})=0 \qquad (4.47)$$

$$\Phi_{2vv}(f+\frac{1}{T})R_{vh}(f+\frac{1}{T})+\Phi_{2vh}(f+\frac{1}{T})R_{hh}(f+\frac{1}{T})=0 \qquad (4.48)$$

$$\Phi_{2hh}(f+\frac{1}{T})R_{hv}(f+\frac{1}{T})+\Phi_{2hv}(f+\frac{1}{T})R_{vv}(f+\frac{1}{T})=0 \qquad (4.49)$$

$$\Phi_{2hh}(f+\frac{1}{T})R_{hh}(f+\frac{1}{T})+\Phi_{2hv}(f+\frac{1}{T})R_{vh}(f+\frac{1}{T})=0 \qquad (4.50)$$

for $0 < f < 1/(2T)$. Again, (f) and $(f-(1/T))$ terms disappeared since they are out of the receiver bandwidth, B_r.

There are twelve equations, equations (4.39) to (4.50), and also twelve unknowns, $R_{vv}(f)$, $R_{vh}(f)$, $R_{hv}(f)$, $R_{hh}(f)$, $R_{vv}(f-\frac{1}{T})$, $R_{vh}(f-\frac{1}{T})$, $R_{hv}(f-\frac{1}{T})$, $R_{hh}(f-\frac{1}{T})$, $R_{vv}(f+\frac{1}{T})$, $R_{vh}(f+\frac{1}{T})$, $R_{hv}(f+\frac{1}{T})$ and $R_{hh}(f+\frac{1}{T})$. Therefore a solution exists for the generalized zero-forcing equalizer to completely suppress ISI, CPI and ACI introduced by two ACI channels under this condition.

From Chapter 3, we have concluded that increased bandwidths have the ability to suppress more interference. This leads to the idea that in ACI analyses, if transmitter bandwidth increases, it might provide the ability to suppress more ACI interferers. The following two examples illustrate some key development of the analyses.

Consider the case where $C = 1.5$, $B_t = 2$, $B_r = B_t$. This is also the overlap case since $2B_t > C$, but there are total of four ACI interferers. It is illustrated in Fig. 4.6. Again, for simplicity, only the vertical-to-vertical and vertical-to-horizontal channels are shown.

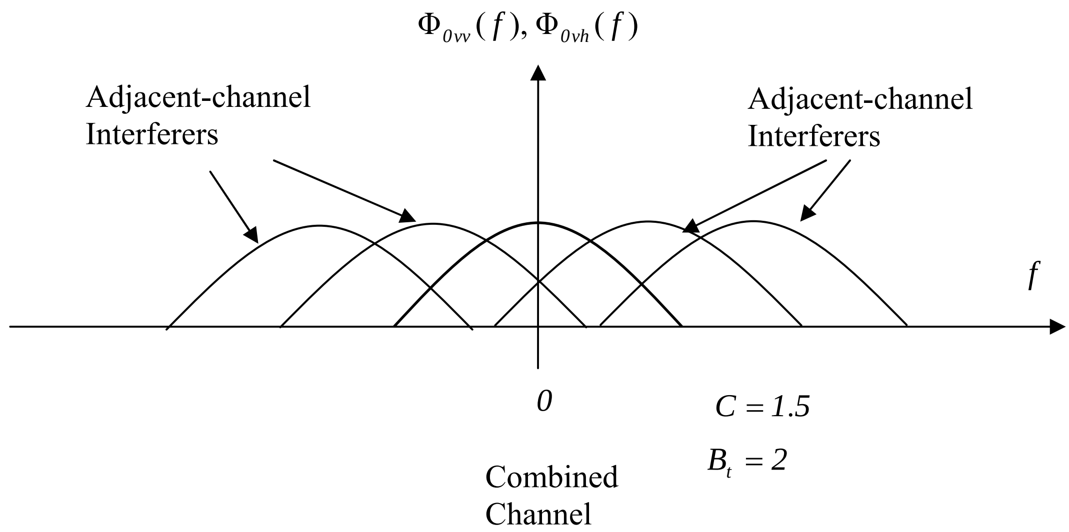


Fig. 4.6 Vertical-to-Vertical and Vertical-to-Horizontal Channel

To completely suppress ISI, CPI and ACI, equations (4.23) to (4.26) reduce to:

$$\Phi_{0vv}(f-\frac{2}{T})R_{vv}(f-\frac{2}{T})+\Phi_{0vh}(f-\frac{2}{T})R_{hv}(f-\frac{2}{T})+\Phi_{0vv}(f-\frac{1}{T})R_{vv}(f-\frac{1}{T})+\Phi_{0vh}(f-\frac{1}{T})R_{hv}(f-\frac{1}{T})+$$

$$\Phi_{0vv}(f)R_{vv}(f)+\Phi_{0vh}(f)R_{hv}(f)+\Phi_{0vv}(f+\frac{1}{T})R_{vv}(f+\frac{1}{T})+\Phi_{0vh}(f+\frac{1}{T})R_{hv}(f+\frac{1}{T})=T$$

(4.51)

$$\Phi_{0hv}(f-\frac{2}{T})R_{vh}(f-\frac{2}{T})+\Phi_{0hh}(f-\frac{2}{T})R_{hh}(f-\frac{2}{T})+\Phi_{0hv}(f-\frac{1}{T})R_{vh}(f-\frac{1}{T})+\Phi_{0hh}(f-\frac{1}{T})R_{hh}(f-\frac{1}{T})+$$

$$\Phi_{0hv}(f)R_{vh}(f)+\Phi_{hh}(f)R_{hh}(f)+\Phi_{0hv}(f+\frac{1}{T})R_{vh}(f+\frac{1}{T})+\Phi_{0hh}(f+\frac{1}{T})R_{hh}(f+\frac{1}{T})=T$$

(4.52)

$$\Phi_{0vv}(f-\frac{2}{T})R_{vh}(f-\frac{2}{T})+\Phi_{0vh}(f-\frac{2}{T})R_{hh}(f-\frac{2}{T})+\Phi_{0vv}(f-\frac{1}{T})R_{vh}(f-\frac{1}{T})+\Phi_{0vh}(f-\frac{1}{T})R_{hh}(f-\frac{1}{T})+$$

$$\Phi_{0vv}(f)R_{vh}(f)+\Phi_{0vh}(f)R_{hh}(f)+\Phi_{0vv}(f+\frac{1}{T})R_{vh}(f+\frac{1}{T})+\Phi_{0vh}(f+\frac{1}{T})R_{hh}(f+\frac{1}{T})=0$$

(4.53)

$$\Phi_{0hv}(f-\frac{2}{T})R_{vv}(f-\frac{2}{T})+\Phi_{0hh}(f-\frac{2}{T})R_{hv}(f-\frac{2}{T})+\Phi_{0hv}(f-\frac{1}{T})R_{vv}(f-\frac{1}{T})+\Phi_{0hh}(f-\frac{1}{T})R_{hv}(f-\frac{1}{T})+$$

$$\Phi_{0hv}(f)R_{vv}(f)+\Phi_{0hh}(f)R_{hv}(f)++\Phi_{0hv}(f+\frac{1}{T})R_{vv}(f+\frac{1}{T})+\Phi_{0hh}(f+\frac{1}{T})R_{hv}(f+\frac{1}{T})=0$$

(4.54)

Four ACI interferers each produce four equations; therefore, the total number of equations is twenty. Note the unknowns are $R_{vv}(f)$, $R_{vh}(f)$, $R_{hv}(f)$, $R_{hh}(f)$, $R_{vv}(f-\frac{1}{T})$, $R_{vh}(f-\frac{1}{T})$, $R_{hv}(f-\frac{1}{T})$, $R_{hh}(f-\frac{1}{T})$, $R_{vv}(f+\frac{1}{T})$, $R_{vh}(f+\frac{1}{T})$, $R_{hv}(f+\frac{1}{T})$, $R_{hh}(f+\frac{1}{T})$, $R_{vv}(f-\frac{2}{T})$, $R_{vh}(f-\frac{2}{T})$, $R_{hv}(f-\frac{2}{T})$ and $R_{hh}(f-\frac{2}{T})$, a total of sixteen. Since the number of unknowns is less than the number of equations, there is no solution.

This example illustrates that the increases of transmitter and receiver bandwidths does not guarantee the complete suppression of interferers. The reason for this is that with the decrease of carrier spacing, more interferers are allowed into the receiver band, therefore even with the increased transmitter and receiver bandwidth, it cannot overcome constrains that the system introduced.

The following example shall reveal the relationships among carrier spacing, transmitter and receiver bandwidths.

Consider the case where $C=2$, $B_t=2.5$, $B_r=B_t$. This is still the overlap case since $2B_t>C$. It is obvious that there are still four ACI interferers introduced.

The unknowns are $R_{vv}(f)$, $R_{vh}(f)$, $R_{hv}(f)$, $R_{hh}(f)$, $R_{vv}(f-\frac{1}{T})$, $R_{vh}(f-\frac{1}{T})$, $R_{hv}(f-\frac{1}{T})$, $R_{hh}(f-\frac{1}{T})$, $R_{vv}(f+\frac{1}{T})$, $R_{vh}(f+\frac{1}{T})$, $R_{hv}(f+\frac{1}{T})$, $R_{hh}(f+\frac{1}{T})$, $R_{vv}(f-\frac{2}{T})$, $R_{vh}(f-\frac{2}{T})$, $R_{hv}(f-\frac{2}{T})$, $R_{hh}(f-\frac{2}{T})$, $R_{vv}(f+\frac{2}{T})$, $R_{vh}(f+\frac{2}{T})$, $R_{hv}(f+\frac{2}{T})$ and $R_{hh}(f+\frac{2}{T})$, a total number of twenty. The number of equations is still twenty since each channel produce four equations. A unique solution exists. Therefore, the generalized zero-forcing equalizer can completely suppress ACI.

This example reveals the fact that with increased transmitter and receiver bandwidth, the carrier spacing also needs to be increased to overcome constrains it might introduced into the system. It means that the generalized zero-forcing equalizer can completely suppress ACI only under certain conditions.

4.4 Calculations Determining Conditions of Zero Interference

The analyses that lead to the ACI results were repeated using the model and the method described in section 4.3. The result is as follows.

The number of interfering ACI signals is:

$$N_{aci} = \begin{cases} 2\,\mathrm{int}(\frac{B_t + B_{ri}}{C}), & C \neq 0, \quad C < 2B_t \\ 0, & C \neq 0, \quad C \geq 2B_t \end{cases} \qquad (4.55)$$

where B_{ri}, the index factor, takes the values:

$$\left\{ \frac{1}{2}, \ \frac{2}{2}, \ \frac{3}{2}, \cdots, \ \frac{\mathrm{int}(2B_r)}{2} \right\}. \qquad (4.56)$$

The function $\mathrm{int}(\bullet)$ denotes the integer part of $\bullet$.

The number of equations is:

$$N_E = 4(1 + N_{aci}) \tag{4.57}$$

The number of unknowns is:

$$N_u = \begin{cases} 0, & 2B_t < 1 \\ 8B_{ri}, & 2B_t \geq 1, C < 2B_t \\ 4\,[\text{int}(2\min(B_{ri}, \quad B_t)) + 1\,], & 2B_t \geq 1, C \geq 2B_t \end{cases} \tag{4.58}$$

where the function $\min(\bullet, \quad \circ)$ denotes the minimum value among $\bullet$ and $\circ$.

Equation (4.55) describes the number of ACI interferers. There are two cases, non-overlap case and overlap case. If it is non-overlap case, $C \geq 2B_t$, then the number of ACI interferers is zero. For the overlap case where $C < 2B_t$, there is a non-zero number of ACI interferers. Equation (4.57) states the calculation of the number of equations and it is easily understood that each channel should produce four equations to satisfy zero ISI, CPI and ACI conditions. The number of unknowns, equation (4.58), has three different cases. If $2B_t < 1$, it is impossible to suppress ISI, therefore the number of unknowns is zero. When overlap occurs, the number of unknowns is determined by the bandwidth B_{ri}. When non-overlap occurs, since the number of ACI interferers is zero, the derivation of the number of unknowns is simply obtained by applying the method in Section 3.3 of Chapter 3.

For the case where the receiver bandwidth is equal to the transmitter bandwidth, the two dimension diagram showing points where a generalized zero-forcing equalizer exists is plotted in Fig. 4.7. The region where ISI, CPI and ACI can be completely suppressed is showing in "+" and "∘" ,where "+" represent the overlap case and "∘" represent non-overlap case, respectively.

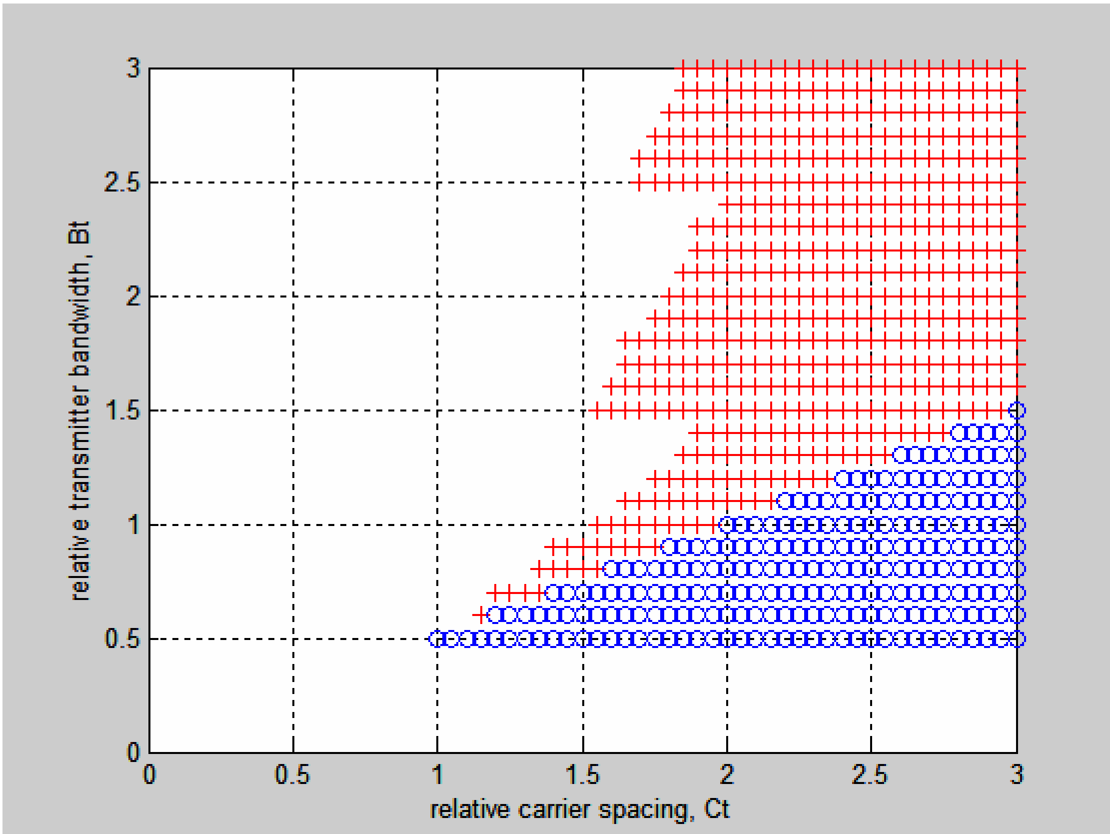


Fig. 4.7 Region of Suppressible ISI, CPI and ACI, $B_r = B_t$

Note that only the lower left boundary of the region is shown in the figure. It is valid if the region extends up to and beyond the upper right corner.

Fig. 4.8 and Fig. 4.9 illustrate the regions of suppressible ISI, CPI and ACI where $B_r = 2B_t$ and $B_r = 3B_t$ are used, respectively.

From Fig. 4.8 and Fig. 4.9, it is suggested that large receiver bandwidths can increase spectral efficiency through decreased carrier spacing.

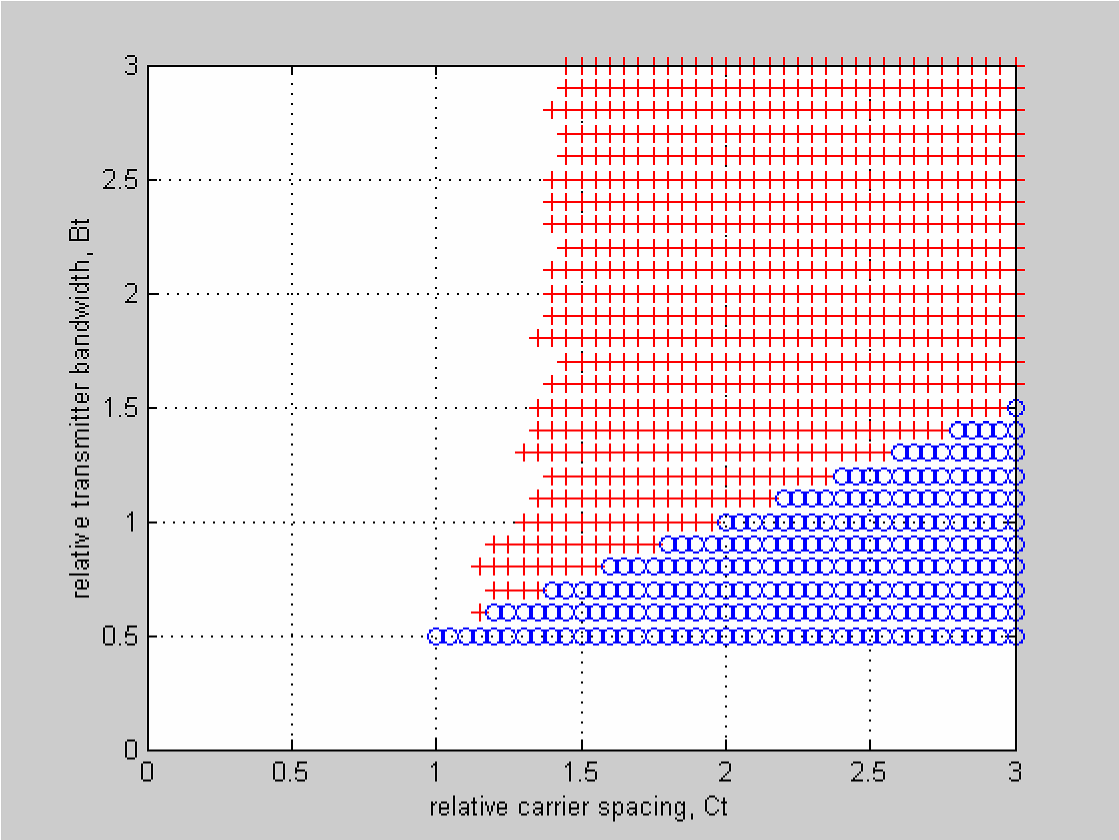


Fig. 4.8 Region of Suppressible ISI, CPI and ACI, $B_r = 2B_t$

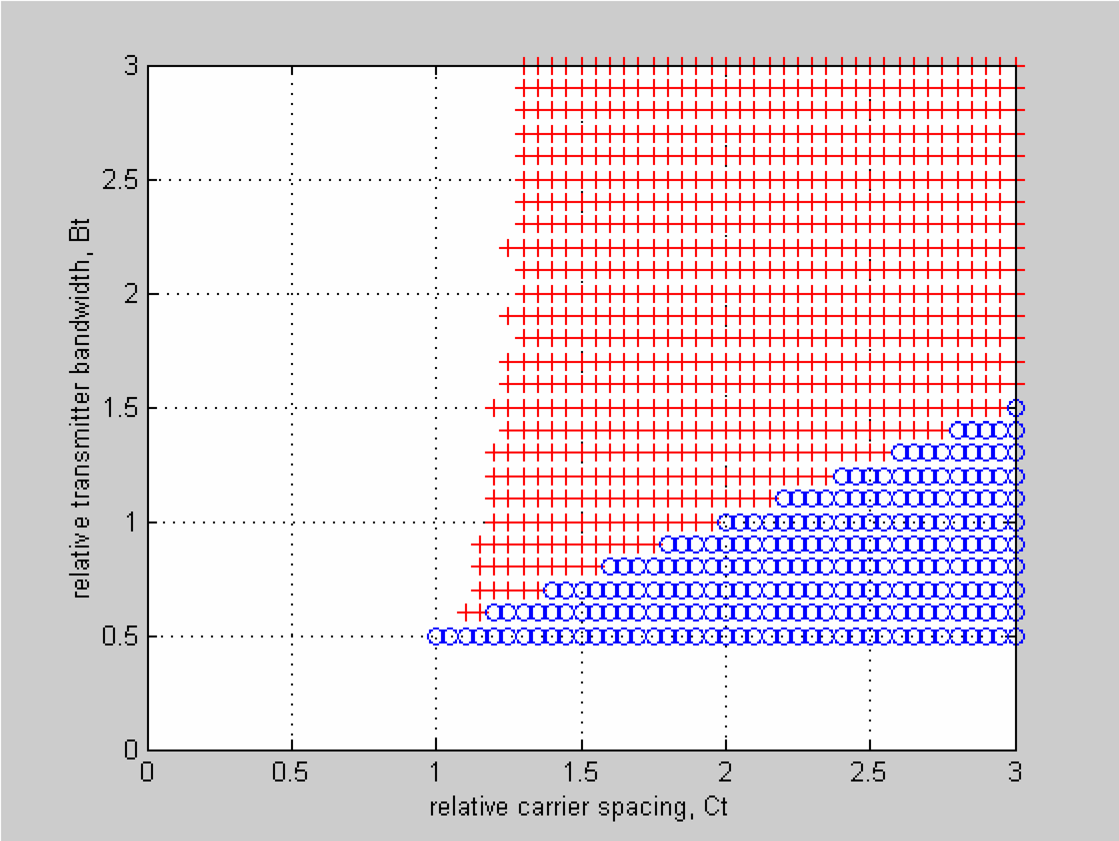


Fig. 4.9 Region of Suppressible ISI, CPI and ACI, $B_r = 3B_t$

Chapter 5

Conclusions

5.1 Conclusions

This thesis has presented the analyses of the abilities to suppress ISI, CCI, CPI and ACI over two polarized fiber-optic channels by generalized zero-forcing equalizers with particular attention given to the effect of bandwidths relative to the symbol rate. The analyses are based on the idea that for the solutions to exist the number of degrees of freedom be greater than the number of constraints. The system operating points could provide guidelines to the system design based on the studies.

An optical fiber communication system model where two 4-QAM signals are transmitted separately over vertical and horizontal polarization channels was constructed. Also, multi-user passband model and baseband model were included.

Based on the presence of ISI, CCI and CPI, the following results are obtained:

It is found that the solutions exist for a generalized zero-forcing equalizer to suppress all ISI, CCI and CPI.

Every increase in transmitter bandwidth, equal to the symbol rate, provides the ability to completely suppress an additional interferer by means of linear equalization. It means if the number of co-channels is L, the combined channel and the combined co-channels are bandlimited to $M/(2T)$, for a solution to exist, the following condition must be satisfied:

$$L \leq M \; . \tag{3.107}$$

where M is from a set of real numbers. This is the fundamental result related to the ISI, CPI and CCI analyses.

Based on the presence of ISI, CPI and ACI, the following results were obtained:

It was discovered that a generalized zero-forcing equalizer has the ability to completely suppress ISI, CPI and ACI. The expressions determining the number of interferers, number of unknowns and number of equations are given by equations (4.55), (4.57) and (4.58), respectively.

The existence conditions which incorporate the effect of transmitter bandwidth, carrier spacing and number of ACI interferers were studied. Under certain conditions, the increase in receiver bandwidth may have the ability to assist the equalizer to suppress more ACI interferers.

5.2 Future Work

The exploration of this thesis are interference analysis and suppression techniques for a fiber-optic communication system operating in the presence of ISI, CPI, CCI, ACI and additive white noise. One important topic for future work is to evaluate the performance through simulation based on the model created in this thesis. Although the study has shown the abilities of interference suppression, how well the system performs under the conditions assumed in the thesis will assist one to evaluate the results discovered. For instance, experiments on equalizer, numerical conditions, and ill-conditioned analyses would be very useful.

Another issue is to apply the results to different types of communications, such as wireless communications and wired communications, and conduct a thorough analysis based on these types of communications.

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